

# A Classical Approach Reveals the Subtle Forces that Modify Newton's Law of Gravity

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**Abstract:** An analogy of Newton's Gravity Law is made with Wilhelm's Weber law of force between two charged bodies, which he introduced in about 1846. An analysis of Weber's Force Law and with the analogy reveals subtle gravity forces that can expand Newton's Gravity force law. Using Weber's law of Force with some modification and as a prototype force formula for gravitation, the perihelion advance of Mercury orbiting around the sun is calculated in a computer program. The calculation results in the same value of approximately 43 arcseconds/century that Einstein calculates from his General Relativity theory. This model offers a classical force-based explanation of relativistic gravitational effects, rather than relying on spacetime curvature.

## Introduction

Andre Assis edited and translated Wilhelm's Weber's work into English where Weber's Force Law can be found [1]. Weber's force law is not acknowledged in today's mainstream physics. This article takes a mathematical approach to examine the law in detail. With the analogy of Weber's force law to the gravity force law, a new gravity law is tested by calculating the perihelion advance of the planet Mercury and comparing the calculated value with the value given by astronomers. It is a provocative idea that relativistic corrections can emerge from classical dynamics.

Many students and researchers struggle with the geometric abstraction of general relativity. This approach provides a mechanical, intuitive framework grounded in energy and motion. It could serve as a pedagogical bridge between Newtonian mechanics and Einstein's theory.

By extending the new gravity force law to dynamic mass flows, you naturally arrive at a wave equation for gravity. This parallels how Maxwell's equations lead to electromagnetic waves. It opens the door to classical modeling of gravitational radiation, potentially useful in education or alternative theory development. However, this aspect is not developed in this article.

## Analyzing Weber's Force Law (1846)

Weber's Force Law between charged bodies:

$$\mathbf{F}_{12} = \frac{kq_1q_2}{r_{12}^2} \left[ 1 - \frac{\dot{r}_{12}^2}{2c^2} + \frac{r_{12}\ddot{r}_{12}}{c^2} \right] \hat{\mathbf{r}} \quad (1)$$

where  $k = 1/4\pi\epsilon_0$  ( $\epsilon_0$  = permittivity of free space) and  $c$  = speed of light,  $q_1$  and  $q_2$  are charge values of charged particles,  $r_{12}$  is the magnitude of the distance between isolated charges  $q_1$  and  $q_2$ ,  $\hat{\mathbf{r}}$  is a unit vector pointing in the direction of the distant vector  $\mathbf{r}_{12}$  between  $q_1$  and  $q_2$ , and  $r_{12}$  is the magnitude of  $\mathbf{r}_{12}$ .

Velocity term derivative:

$$\dot{r}_{12} = \left| \frac{d\mathbf{r}_{12}}{dt} \right| \quad (2)$$

Acceleration term derivative:

$$\ddot{r}_{12} = \left| \frac{d\mathbf{v}_{12}}{dt} \right| \quad (3)$$

$\mathbf{f}_q$  is defined as the Coulomb force vector and  $f_q$  is defined as the Coulomb law:

$$\mathbf{f}_q = f_q \hat{\mathbf{r}}_{12} = \frac{kq_1q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}, \quad (4)$$

Expanding (1) with  $f_q$ :

$$\mathbf{F}_{12} = f_q \hat{\mathbf{r}}_{12} - \frac{f_q \dot{r}_{12}^2 \hat{\mathbf{r}}_{12}}{2c^2} + \frac{f_q r_{12} \ddot{r}_{12} \hat{\mathbf{r}}_{12}}{c^2} \quad (5)$$

Each term in (5) is a force formula and the directions of the forces are in line with the unit distance vector  $\hat{\mathbf{r}}_{12}$ . The first term on the right side of (5) is Coulomb's Law. The second term on the right side of (5) is a force term with velocity squared. The third term on the right side of (5) is a force term with acceleration. We observe in (5) that the velocity term and the acceleration term change the overall force value  $\mathbf{F}_{12}$  of Weber's force law.

Modern physics defines  $E = mc^2$  or  $m = E/c^2$ . This equation is thought to have been derived from special relativity (SR) [2]. However, its first use appeared with Weber since he derived the speed of light  $c$ .  $m = E/c^2$  will appear in the velocity and acceleration terms of (5) as will be shown next.

Acceleration term:

The Weber's force law (1) does not restrict the size of masses  $m_1$  and  $m_2$ , of the charges  $q_1$  and  $q_2$ . The potential mass  $m_p$  created by the force between the charges is:

$$m_p = \frac{E}{c^2} = \frac{f_q r_{12}}{c^2} \quad (6)$$

In the Weber Law a potential energy mass is created from the energy derived from force times distance energy. If one of the masses or both of the masses of the charges is greater than the masses created by the energy of  $f_q r_{12}$  as shown in (6), the formula is not concerned about the excess. Weber derived the velocity of light  $c$ , but I think he did not know at that time it was the velocity of light! Substituting (6) into the acceleration term of (5), the acceleration force term which we will define as  $\mathbf{f}_a$  becomes:

$$\mathbf{f}_a = m_p \ddot{r}_{12} \hat{\mathbf{r}}_{12} \quad (f = ma) \quad (7)$$

This formula (7) shows that potential energy mass effects the basic force between the charges. Also, the formula  $f = ma$  is a common Newton force law.

Velocity term:

The vector force for the velocity term of (5) is defined as  $\mathbf{f}_v$ . Substituting (6) in the velocity term:

$$\mathbf{f}_v = \frac{f_q \dot{r}_{12}^2 \hat{\mathbf{r}}_{12}}{2c^2} = \frac{f_q r_{12} \dot{r}_{12}^2 \hat{\mathbf{r}}_{12}}{2c^2 r_{12}} = \frac{m_p \dot{r}_{12}^2 \hat{\mathbf{r}}_{12}}{2r_{12}} \quad (8)$$

In (8) the velocity term is a classical force term. It is a force term if the velocity vector of  $\dot{\mathbf{r}}$  is at right angle with respect to  $\hat{\mathbf{r}}_{12}$ . Using polar coordinates, temporarily, keeping  $r$  constant:

$$\dot{\mathbf{r}} = r\omega = v \quad (9)$$

where  $\omega$  is the angular velocity of vector  $\mathbf{r}$ . Note that  $\dot{\mathbf{r}}$  is the relative velocity between the two charges and this relative velocity can be perpendicular to the vector  $\hat{\mathbf{r}}_{12}$ .

Substituting (9) into the term on the right side of (8):

$$\mathbf{f}_v = \frac{m_p v^2 \hat{\mathbf{r}}_{12}}{2r_{12}} = \left( \frac{f_q r_{12}}{c^2} \right) \frac{v^2 \hat{\mathbf{r}}_{12}}{2r_{12}} \quad (10)$$

$\mathbf{f}_v$  is a centrifugal force repelling the charged bodies.

Some features of Weber's Force Law:

1. The force law, describing a force between two charges, adds two inertial forces created by the potential energy mass, without an obvious external force acting on the two charges
2.  $\mathbf{F}_{12} = \mathbf{f}_q - \mathbf{f}_v + \mathbf{f}_a$  if no external force or energy is added to the system of the two charges. The formula appears to be a combination of Newtonian physics and relativistic physics.
3. The relative velocity and acceleration of the potential energy masse effects the force between the charges.
4. The law embodies centrifugal force, Newton's second law,  $f = ma$ , and special relativity with  $m = E / c^2$ .

### Modification of the velocity term in (1)

Before we make by analogy of the Weber Law to a gravity law, the Weber law needs some changes. The first modification is to make the velocity value used in the velocity term be a component value perpendicular to  $\mathbf{r}_{12}$ : Define an angle  $\theta$  to be the angle between the velocity vector  $v$  and the distance vector  $\mathbf{r}_{12}$ . Depending on whatever direction the velocity vector is pointing, multiply the velocity magnitude term  $v$ , with sine  $\theta$ :

$$(v \sin \theta)(v \sin \theta) = v^2(1 - \cos^2 \theta) \quad (11)$$

Including (11) into (10):

$$\mathbf{f}_v = \left( \frac{f_q r_{12}}{c^2} \right) \frac{v^2 (1 - \cos^2 \theta) \hat{\mathbf{r}}_{12}}{2r_{12}} \quad (12)$$

The second modification relates to the kinetic energy of the potential energy mass. This kinetic energy of the potential energy mass also creates a mass. This mass adds to the potential energy mass. When it moves perpendicular to the distance vector  $\mathbf{r}_{12}$  it creates a centrifugal force. The derivation of a velocity term force based on kinetic energy of the potential energy mass:

$$\frac{m_p v^2 (1 - \cos^2 \theta) \hat{\mathbf{r}}_{12}}{2r_{12}} = \left( \frac{f_q r_{12}}{c^2} \right) \frac{v^2 (1 - \cos^2 \theta) \hat{\mathbf{r}}_{12}}{2r_{12}} \quad (13)$$

Adding (13) to the velocity term of (12) the "2" is eliminated in the denominator of the velocity term in the Weber force.

The modified Weber force law is now:

$$\mathbf{F}_{12} = \frac{kq_1q_2}{r_{12}^2} \left[ 1 - \frac{\dot{r}_{12}^2(1 - \cos^2 \theta)}{c^2} + \frac{r_{12}\ddot{r}_{12} \cos \theta}{c^2} \right] \hat{\mathbf{r}} \quad (14)$$

Also, we include a  $\cos \theta$  factor in the acceleration term to insure the component of the acceleration vector is in line with  $\mathbf{r}_{12}$ . ( For computer program  $\ddot{r} \cos \theta \Rightarrow (v_2 \cos \theta_2 - v_1 \cos \theta_1) / dt$  )

### Consideration of the sign of the Weber force

Eq. (14) sign is set for charges of like sign. A positive resultant force represents repulsion. If we are going to make (14) work for gravity force which represents attraction, then we need to change the signs of the terms in the bracket of (14):

$$\mathbf{F}_{12} = \frac{kq_1q_2}{r_{12}^2} \left[ -1 + \frac{\dot{r}_{12}^2(1 - \cos^2 \theta)}{c^2} - \frac{r_{12}\ddot{r}_{12} \cos \theta}{c^2} \right] \hat{\mathbf{r}} \quad (15)$$

### Applying Weber's modified force law for gravity force by analogy of mass to charges

In (15) substitute  $M_1$  and  $m_2$  for the charges with  $M_1 \gg m_1$ . Also, substitute  $G$  for  $k$ . For simplicity neglect the orbit of  $M_1$  and have  $m_2$  orbiting  $M_1$  in a circular orbit with velocity  $v$ . Writing out the Weber equation for these conditions:

$$\frac{GM_1m_2}{r_{12}^2} \left[ -1 + \frac{v^2(1 - \cos^2 \theta)}{c^2} - \frac{r_{12}}{c^2} \frac{(v_2 \cos \theta_2 - v_1 \cos \theta_1)}{dt} \right] = \frac{m_2v^2}{r_{12}} \quad (16)$$

where  $\theta$  remains the angle between the velocity vector  $\mathbf{v}$  and the radial vector  $\mathbf{r}_{12}$ . Note that the velocity term of (15) represents centripetal force of the mass created by potential energy mass of the force relation, not the total mass of  $m_2$ . Also, note that the velocity direction must be perpendicular to the radial vector  $\mathbf{r}_{12}$  as given by the  $(1 - \cos^2 \theta)$  factor or the equivalent  $(\sin^2 \theta)$  factor.

Since the kinetic energy of  $m_2$  increases its inertial mass we have:

$$m_2^k = m_2 + \frac{m_2v^2}{2c^2} = m_2 \left( 1 + \frac{v^2}{2c^2} \right) \quad (17)$$

Including (17) in (16):

$$\frac{GM_1m_2}{r_{12}^2} \left( 1 + \frac{v^2}{2c^2} \right) \left[ -1 + \frac{v^2(1 - \cos^2 \theta)}{c^2} - \frac{r_{12}}{c^2} \frac{(v_2 \cos \theta_2 - v_1 \cos \theta_1)}{dt} \right] = \left( 1 + \frac{v^2}{2c^2} \right) \frac{m_2v^2}{r_{12}} \quad (18)$$

The factor modifying  $m_2$  on the right and left side of (18) is like a gamma factor in SR for low velocities compared to  $c$ . No matter how much  $m_2$  increases due to potential and kinetic energy mass increase, its gamma factor cancels out on both sides of (18). Mercury's perihelion advance is calculated in a Pascal program using (16) to test the validity of (16).

### Calculating procedures for Mercury perihelion advance

Use the excess force terms of the gravity force law of (16) to calculate the difference from a pure elliptical orbit defined by the Newtonian Gravity Law. These terms are the velocity term and the acceleration term of (16). These force deviations cause the orbiting mass to deviate from a pure

elliptical path and create the perihelion advance. We define, using polar coordinates [3], a pure elliptical path for Mercury from the astronomical data for Mercury [4]. Divide the orbital path of Mercury into a number of equal rotational angles of the radius from the focus, the sun, to mercury. In this case use  $\pi/20$  for the angle increment. That divides one orbit into 40 equal angle segments. Calculate the radius, the velocities for each segment, and the time of Mercury's travel for each segment. The velocity perpendicular to the radius at each segment is also calculated [5]. The excess force calculated from the force velocity term and the acceleration term when applied to Mercury at each segment causes Mercury to deviate from its pure path. The amount it deviates divided by the radius at each segment is the perihelion advance for that segments. These advances are totaled for 100 Earth years producing the total amount of the perihelion advance. Appendix A is a copy of the pascal program that accomplishes all this.

## Results

**Table 1. Perihelion advance arc-sec per 100 Earth years.**

Planet	Weber-Gravity Force Law (Modified)	General Relativity (GR)*	Einstein quoting astronomers
Mercury	43.9350	42.98	45 <sup>+/-5</sup>
Venus	9.2588	8.62	
Earth	4.1071	3.84	
Notes	MERSRWEB.PAS Pascal Program	*Wikipedia	**See quote below table

\*\*Here is a quote from Einstein's paper of 1915 when he discloses his application of GRT to the Mercury perihelion advance: "This calculation leads to the planet Mercury to move its perihelion forward by 43" per century, while the astronomers give 45"±5", an exceptional difference between observation and Newtonian theory. This has great significance as full agreement." [6]

## Discussion

1. The Modified Weber Force formula when applied to calculating Mercury's perihelion advance produces the right answer. It is Newtonian with exception of the use of  $E = mc^2$ .
2. The unique feature of Weber's force law forces one to account for the centrifugal forces created by movement of the mass created by the potential energy interaction. This explains effects we attribute to GR.
3. The modified Weber force formula computer calculates a more accurate answer to the perihelion advance of 45<sup>+/-5</sup> provided by the astronomers quoted by Einstein.
4. The modified Weber's force law applies to celestial bodies.

## Conclusion

1. Weber has written the first relativistic equation long before Einstein's SR and GR.
2. The force laws of physics, somewhat hidden in GR are revealed in Weber's Force-Gravity Law. These force laws are derived from the mass created by the potential energy. They are the centrifugal force of the right-angle velocity of the potential mass, and the centrifugal force created by right- angle velocity associated with the kinetic energy of the potential energy mass,

and the relative acceleration of the potential energy mass in line with the distance between the masses.

3. Eq. (16) has many useful applications for relationship of celestial bodies. With the exception of  $E = mc^2$ , all terms in the Weber-Gravity Force formula appears to be classical. This suggests a 3D space.
4. It is a provocative idea that relativistic corrections can emerge from classical dynamics.

## References

- [1] A. K. T. Assis (editor), "Wilhelm Weber's Main Works on Electrodynamics Translated into English", Volumes 1-5. <https://www.ifi.unicamp.br/~assis/books.htm>
- [2] Prof Donald C. Chang, "New research on the physical basis of mass–energy equivalence", <https://www.youtube.com/watch?v=vVaPBicrgFY>
- [3] The Chemical Rubber Co., Handbook of Mathematical Tables, Second Edition, Supplement to Handbook of Chemistry and Physics (1964), Planetary Orbits, Friend H. Kierstaed, Jr., pgs. 501-509.
- [4] CRC, Handbook of Chemistry and Physics, 52<sup>nd</sup> Edition (1971-1972), pgs. F-145-150
- [5] J.J Tuma, PhD, Engineering Mathematics Handbook, 2<sup>nd</sup> Edition, McGraw-Hill Book Company, (1979) p.69.
- [6] Einstein's Paper: "Explanation of the Perihelion Motion of Mercury from General Relativity Theory", Anatoli Andrei Vankov, IPPE, Obninsk, Russia; Bethany College, KS, USA; [anatolivankov@hotmail.com](mailto:anatolivankov@hotmail.com).

## Appendix A

Program Mersrweb; {Modified Weber's force law with acceleration term applied to gravity force.}

{ $N+$ }

Uses CRT;

Const

c = 2.997925E+8;      {Velocity of light}  
 vp = 58.921e+3;      {Velocity of Mercury at perihelion}  
 va = 38.824e+3;      {Velocity of Mercury at aphelion}  
 m2 = 3.181e+23;      {mass of Mercury}  
 m1 = 1.991e+30;      {mass of sun}  
 G = 6.67259E-11;      {Gravitational constant}  
 rp = 4.604e+10;      {Radius at perihelion}  
 ra = 6.986e+10;      {Radius at aphelion}  
 a = 5.795e+10;      {distance from center of ellipse to perigee}  
 {ec = 0.2056;}  
 Tdays = 87.97;      {period of Mercury orbit in earth days}

Var

i,j : integer;      {incrementing integers}

ec,d,            {calculated basic elliptical parameters}  
 r,                {distance between focus and Mercury at  
                   end of a segment}  
  
 z,                {dummy variable}  
 x,y,dx,dy,        {space variables}  
 t,                {time of m2 travel in a segment}  
 Tc,                {cumulative time}  
 Torb,             {time for one orbit}  
  
 sigma,dsigma,        {angle between r and x-axis}  
 fi,                {angle between Vx and V}  
 theta,theta1,theta2,   {angle between r and V}  
 mu,                {G\*m1}  
  
 vx,                {velocity of m2 perpendicular to r at r}  
 v,                {actual velocity of m2 at r}  
 v1,v2,             {velocity at beginning and end of segment}  
 vavg,             {average velocity of a segment of calculation}  
 vm,                {slow moving velocity of Mercury}  
 dadv,             {a segment perihelion advance,sec}  
 adv,              {summed dadv of all segments for one orbit}  
 vrm,arm,Frm,        {intermediate variables to simplify formulas}  
 totalsec:real;      {perihelion advance}

Begin

```

Clrscr;
{Ellipse parameters}
ec := 1.0 - rp/a;
Torb := Tdays*24.0*60.0*60.0;
mu := G*m1;
d := 1 + m2/m1;

```

{Initial Conditions}

```

v1 := vp;
vm := 0.0;
sigma:= 0.0;
Tc := 0.0;
Theta1 := Pi/2.0;
dsigma := Pi/20;

```

{calculation for full ellipse}

```

i := 1;

```

```

repeat
  sigma := sigma + dsigma;

  {Calculate r}
  if i = 20 then r := a*(1+ec) else
  r := a*(1.0 - ec*ec) / (1.0 + ec*cos(sigma));

  {calculate v, actual velocity of m2}
  v := sqrt(mu*((2.0/r) - (1.0/a)));
  v2 := v;

  {calculate vx, m2 velocity perpendicular to r}
  z := mu*a/(r*r);
  vx := sqrt(z*(1.0 - ec*ec));

  {Calculate vavg for each segment}
  vavg := (v2 + v1)/2.0;

  {calculate t, time for each segment}
  t := r*dsigma/vavg;

  {Calculate the angle theta to use in the cosine term of the SR and Weber formulations}
  y := Vx/v;
  fi := arctan(sqrt((1.0 - y*y))/y);  {always + value!}
  theta := Pi/2 - fi;  {whether sign of fi is + or -, you get same answer with square of cos(theta)}
  theta2 := theta;
  frm := G*m1/(r*d*r*d);  {acceleration a=Force/m2}
  vrm := v*v*(1.0 - cos(theta)*cos(theta))/(c*c);
  arm := r*(v2*cos(theta2)-v1*cos(theta1))/(t*c*c);

  dadv := frm*(+vrm -arm)*t*t/r + vm*t/r;
  adv := adv + dadv;
  vm := frm*(+vrm - arm)*t + vm;

  v1 := v2;
  theta1 := theta2;
  i := 1 + i;
  Until (i = 41);  {end i loop}

  {Calculate the perihelion advance}
  totalsec :=adv*(180.0/Pi)*3600.0*(36525/Tdays);
  writeln('totalsec = ',totalsec);
  readln;
End.

```