Wave Model of the Proton and Electron

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Abstract

While writing this paper I discovered the quantum number *n* expands the radius of a particle's size and not so much its mass-energy. I develop the Planck force formula and apply it to the wave model of the electron and the proton. The model has two sub-particles in each of the electron and proton particles. The two sub-particles are viewed as orbiting each other at near the speed of light forming a standing wave. Also, it was discovered that the Planck force has intrinsic relativistic properties such that relativity terms as gamma factors do not have to be applied in the equations [1]. This allows for simpler math equations for the model.

Introduction

This model is based on the de Broglie light wave formula and the physics of Plank's Force. One physicist [2] has already written about a similar proton model and another [3] has suggested it with her OM model. I describe a more complete math model. The key to unlocking the math related to this model is the accurate measured radius, r_p of the proton which is 8.412 x 10-16 m. That was reported by Arend Niehauas's [2]. His partial math model of the proton is:

$$r_p = \frac{4\hbar}{m_p c} = 8.4123 \times 10^{-16} \text{ m} \quad (1)$$

where r_p is the measured radius of the proton, m_p is the mass of the proton, *c* is the speed of light, and $\hbar$ is reduced Plank's constant, $h/2\pi$. His formula predicts the measured value quite well. The fact the '4' in the numerator of (1) is an integer is a clue to the correctness of (1). (1) is descriptive of the proton in a de Broglie wave. The electron is similarly modeled as a de Broglie wave.

Preliminary consideration of light characteristics

The energy and wavelength of a light wave is given by the energy and *de Broglie* relation:

$$E = hf = h/T \quad (2)$$

$$\lambda = h/\rho \quad (3)$$

where *E* is energy, *h* is Planck's constant, *f* is the frequency of the light wave, *T* is the period, λ is the wavelength of the light wave and ρ is the momentum of the light wave. I postulate that as light travels through space, it creates mass bit charges as it travels. These bit charges are left behind, annihilated, and only the energy of the light is transported through space. (A Maxwell equation has the electric field created by the rate of change of the magnetic field of the light wave instead of charges being created to create the electric field.) In a electron or proton particle the energy of a light wave is captured as a standing wave and the mass bits of the standing wave become observable.

Wavelength of material particle

The wavelength of a particle is reported to be [4]:

$$\lambda = h/\rho = h/mv \quad (4)$$

Where *m* is the mass of the particle and *v*, generally *c* for particle description, is its velocity relative to a CM reference point. A material particle can be considered to be a standing wave whereas a light wave or "photon" is considered to be traveling linearly at speed *c*. When we regard a particle as a wave that moves in a circular orbit, stationary waves in the orbit can exists when the circumference $2\pi r$ of the orbit is a n integer or half integer multiple of the particle's wavelength:

$$n\lambda = 2\pi r \tag{5}$$

where n = .5,1,1.5.2,2.5,3,3.5,etc.
Substituting (4) into (5):

$$mvr = n\hbar \tag{6}$$

Two charged mass bits of a particle orbiting each other creating the electron or proton

The way this wave-particle theory works is to think of either the electron or proton as being composed of two charged mass bits of opposite charge and equal mass. Each charged mass bit does need to have spin associated with it, but it may have. The mass of each bit is the relativistic mass of each bit orbiting the other near the speed of light. The relativistic mass of each mass bit is one-half the mass of the whole particle as viewed from the CM frame of reference of the electron or proton, the same frame of reference an external observer has.

The fine structure constant *α*

While playing with numerical values of know constants, I discovered the follow equalities are true:

$$\alpha = \frac{r_e m_e c}{\hbar} = \frac{r_e m_e c^2}{\hbar c} = \frac{ke^2}{\hbar c} = \frac{\text{Coulomb force}}{\text{Plank force}} = \frac{1}{137} \tag{8}$$

where r_e is the usual electron radius and m_e is the mass of the electron:

$$r_e = ke^2 / m_e c^2 = 2.817940287723 \times 10^{-15} \text{ m} \tag{9}$$

where *k* is Coulomb's constant $1/4\pi\varepsilon_o$, ε_o is the permittivity of free space, and *e* is the electric charge.
As a side note, rearranging (8):

$$r_e = \frac{\alpha\hbar}{m_e c} \tag{10}$$

Note the similarity of (10) to (1) and (6).
From (9) Coulombs law may be expressed as follows:

$$\frac{ke^2}{d^2} = \frac{r_e m_e c^2}{d^2} \tag{11}$$

where *d* is the distance between the two charges. We note that the term on the right of (11) contains no observable factor of the permittivity and permeability. The permittivity of free space is intrinsic in the calculation of r_e , see (9), and I assert that all following calculations involving r_e have the permittivity of free space. We show (8) as force laws:

$$\frac{ke^2}{\alpha d^2} = \frac{r_e m_e c^2}{\alpha d^2} = \frac{\hbar c}{d^2} \tag{12}$$

Plank's Force law is the term on the right of (12). Its force is about 137 times the Coulomb force at the same distance. We can expect the Plank's force law to be the force between two particles since the two terms on the left of (12) are force laws between two charges or particles.

The mass-energy of electron-positron annihilation goes to zero at distances less than $r_e/2$. So, there is likely to be no Coulomb force between two charged particles with separation distance less than $r_e/2$. To reflect force between internal sub particles we will employ the Plank force law. Also, we will assume the potential masses created by the force between the sub particles are the only masses of the whole particle. I consider the augmented Planck force term and its corresponding energy term divided by c^2,

$m = E/c^2$ are sufficient for relativistic considerations. *That idea is key to understanding this model.* The mass of the electron and the proton particle are given by the general formula:

$$m_{pariicle} = \frac{\hbar c}{d_{particle} c^2} = \frac{\hbar}{d_{particle} c} \tag{13}$$

While I do not derive the mass of the electron, I consider it legitimate to use the measured mass of the electron and the proton in this paper to represent the relativistic masses of the mass bits, created by potential energy, traveling near the speed of light.

The separation of the two-bit masses of comprising the particle is given by the general formula:

$$d_{particle} = \frac{\hbar}{m_{particle} c} \tag{14}$$

Notice the similarity of (14) to (1) and (6)!

The $d_{particle}$ is considered by Lori Gardi to be the radius r of the wavelength of the particle. She sets $2\pi r$ as the wavelength λ of the particle. I set $r' = r/2 = d/2$ as the radius of the orbiting bit masses, but this radius is not considered primary to the standing wavelength of the particle.

The electron particle

Electron radius:

From (14):

$$r = d = \frac{\hbar}{m_e c} = 3.861593 \times 10^{13} \text{ m} \tag{15}$$

If we multiply the radius *r* in (15) by $1/\alpha$ we get A = 5.291772 x 10^{-11} m, the radius of the Bohr orbit in a hydrogen atom. If we multiply the radius in (15) by α ,see (10), we get r_e .

Electron wavelength:

Computing the electron wavelength λ :

$$\lambda = 2\pi r = 2.426310 \times 10^{-12} \text{ m} \tag{16}$$

Electron spin:

Equating the Planck force to the centripetal force of one of the two masses comprising the electron:

$$\frac{\hbar c}{d^2} = \frac{m_e c^2}{2(d/2)} \tag{17}$$

$$\frac{\hbar}{d} = m_e c \tag{18}$$

With $2r' = d$ and solving for angular momentum in(17):

$$\text{Spin} = m_e c r' = \frac{\hbar}{2} \tag{19}$$

We note this value of spin is what is given in textbooks for the spin of the electron.

Also, we note the orbital angular momentum of the two orbiting masses of the electron add to make $\hbar/2$.

Also, note that $r' = \frac{d}{2} = \frac{r}{2}$.= 1.930796 x 10^{-13} m.

Electron rotational frequency f_e :

Since it takes two rotations to make one electron wavelength, from (19):

$$f_e = \frac{1}{T} = \frac{c}{2\pi r'} = 2.471179951 \times 10^{20}\ \text{Hz} \quad (20)$$

Electron magnetic moment:

The derivation the magnet moment μ for the electron configuration presented by this model follows. Note the differences with the standard model of the electron. We start with $\mu = iA$ where *i* is current and *A* is the area enclosed by the current loop. The current loop is $2\pi r'$ in length; the area $A = \pi r'^2$ where *r′* is the rotation radius of the electron masses. *T* is the time required for a mass in the model to traverse one loop length. $T = 2\pi r'/c$; $i = \pm e/T$; putting these together: $\mu = ecr'/2$. For one of the two masses in the model $r' = \hbar / 2m_e c$. Combining the last two equations: $\mu = \pm e\hbar / 4m_e$.(Both masses in the electron particle produce a total charge of *e*.) The value physics gives to the magnet moment of the electron is −9.2847646917(29)×10^{-24} J·T^{-1}. The Bohr Magnetron is given by:

$$\mu_B = \frac{-e\hbar}{2m_e} = -9.274009 \times 10^{-24} \quad (21)$$

Which is twice the $\mu = \pm e\hbar / 4m_e$ predicted above for this model.

The electron intrinsic spin magnetic moment is given by:

$$\mu_s = -g_s \mu_B \frac{S}{\hbar} = -g_s \mu \quad (22)$$

S is the electron spin angular momentum and g_s is the spin g-factor. g_s is approximately equal to 2. μ_s and S are vectors. g_s = 2.00231304 spin g_s factor. $a_e = \alpha / \pi$ = 0.002322819 contributes to anomalous magnetic moment. The fact that the spin g_s factor has a value of approximately 2 may be related to the fact that it takes two rotations of the masses to make one wavelength of the electron.

Electron charge:

We note that by equating the numerators of the left two terms of (10) that:

$$e^2 = r_e m_e \times 10^7 \quad (23)$$

The positive and negative charges orbiting each other in this model will cancel each other making the overall charge of the electron zero. So, there must be some other factor in the model which creates charge. We might speculate that it the intrinsic spin angular momentum that creates charge since the spin angular momentum of the electron matches that of the proton. Angular momentum is conserved in the manner charges are conserved. Also, the spin angular momentum of the two masses add in the model instead of subtracting like the charges. So, this suggests that the intrinsic spin angular momentum of a particle defines its charge. If this is the case, a charge particle may be either + or – depending on its orientation with respect to another particle. ($\hbar / 2$ seems to be, but not necessarily, the key spin angular momentum for a charge of *e*.)

Velocity of mass-bits:

By setting $\gamma = 1/\alpha = 1/\sqrt{1 - v^2/c^2}$ and solving for *v*. $v = c\sqrt{1-\alpha^2}$ = 2.997844757 x 10^8 m/s (almost the speed of light)

The proton particle

Proton radius:
From (14):

$$r = d = \frac{\hbar}{m_p c} = 2.103089 \times 10^{-16} \text{ m} \qquad \{24\}$$

Applying (6), $mvr = n\hbar$, with n = 4:

$$r_p = \frac{4\hbar}{m_p c} = 8.4123 \times 10^{-16} \text{ m} \qquad (25)$$

(25) agrees with (1) which we set out to demonstrate. The fact that the value of the measured radius of the proton agrees with the results of (25) supports the wave model of the proton and the idea a proton can have different quantum states.

Proton wavelength:
Computing the proton wavelength λ:

$$\lambda = 2\pi r = 1.3214010^{-16} \text{ m} \qquad (26)$$

Proton spin:
We employ Plank force law and centripetal force of one of the masses in the proton:

$$\frac{\hbar c}{d^2} = \frac{m_p c^2}{2(d/2)} \qquad (27)$$

$$\frac{\hbar}{d} = m_p c \qquad (28)$$

With $2r' = d$ and solving for angular momentum in(28):

$$\text{Spin} = m_p c r' = \frac{\hbar}{2} \qquad (29)$$

We note this value of spin is what is given in textbooks for the spin of the proton.
Also, we note the orbital angular momentum of the two orbiting masses of the proton add to make $\hbar / 2$.

Proton rotational frequency f_p:
It takes two rotations to make one proton wavelength, from (29):

$$f_p = \frac{1}{T} = \frac{c}{2\pi r'} = 4.537464 \times 10^{+23} \text{ Hz} \qquad (30)$$

Proton magnetic moment:
The proton magnetic moment associated with proton spin is measured in units of the *nuclear magnetron* β_I, which is defined as:

$$\beta_I = \frac{e\hbar}{2m_p} = (5.05050 \text{ +/- } 0.00013) \times 10^{-27} \text{ J/(Wb/m}^2) \qquad (31)$$

Where m_p, the proton mass, replaces the electron m_e in the Bohr magnetron. Since the proton mass is 1836.15 greater than the mass of the electron, the nuclear magnetron is that much smaller.
The nuclear magnetic moment of the proton is found by experiment to be:

Proton magnetic moment = +(2.79276 +/- 0.00002) β_I.

By analogy with the electron we can expect the proton's spin magnetic moment to be:

$$\mu_{sp} = g_{sp}\mu_I \frac{S}{\hbar} \tag{32}$$

Substituting in (32) values for $\mu_I = \beta_I$ and spin $S = \hbar / 2$, we find that g_{sp} Should equal 2.000. Instead we find it equal to 2.79276 by experiment. It suspected the extra value of 0.79276, added to 2, is related to factor of 4 in (20). More work is needed here. The nuclear magnet moment for the proton is derived with the same considerations for the electron in this model.

Proton charge:

For this model of the proton to have a magnet moment as described and a charge of +*e* then it is necessary to consider the proton charge as originating from it spin angular momentum $\hbar / 2$ just as for the electron.

The neutron particle

The neutron particle is modeled just like the proton particle. I will propose it is just a proton as described above with a relativistic electron orbiting around it.

Mass Bits

The mass of the two-bit masses comprising the electron or the proton are created by the Planck force potential energy divided by c^2:

$$m = \frac{\hbar c}{dc^2} \tag{33}$$

I speculate that electrons, and positrons are created out of the medium.

Discussion

Arguments for the model:

The two charged mass bit particles form the foundation for this model. One can argue that two charged particles of opposite polarity must exist in a light photon or wave for there to exist an electric field in the photon. Same argument for a particle. An isolated charged particle usually does not exist alone. Since vast clouds of hydrogen, composed of a protons and electrons, exist in the space of the universe, stretching for thousands of light years, we can expect the proton and electron are created out of the medium and at the same time.

I argue the equalities of (8), (17), and (27) are true because they are equal when the numerical values of the constants are inserted into them. These equalities give size and structure to the particles. Each particle has a basic radius and wavelength. But as a wave it can have different radius associated with the *n* quantum state. The fact that the experimental value of radius of the proton agrees with the calculated value of the radius when *n* = 4, as a whole integer, is inserted into the wave equation, is basic support for the wave model.

Arguments against the model:

To my knowledge, angular momentum does not create electric charge. Electric charge does not naturally emerge from (17) or (27). The anomalous magnetic moments of the electron and proton do not emerge from these formulas. They need to be explained. Also, a model for the neutron may be too big.

References

[1] Keele, James, Calculating Perihelion Advance of Mercury Using a Slightly Modified Weber's Force Law (3rd Edition), October 2025. This paper can be downloaded from my website: www.srelectrodynamics.org

[2] Arend Niehauas, An Alternative Model of Proton and Neutron, Journal of Modern Physics > Vol.11 No.2, February 2020 , (Arend Niehaus, Retired, Utrecht University, Utrecht, The Netherlands.)

[3] Lori Gardi, https://www.youtube.com/watch?v=En34H-buYoo theOMparticle ; On Planck Natural Units and the Schwarzschild Radius: BREAKTHROUGH

[4] Weidner, Richard T; Sells, Robert L., Elementary Modern Physics, Second Edition, Allyn and Bacon, Inc. Boston (1973) p.144.

Units and Constants:

RMK Units

Symbols:

- m mass
- v velocity with respect to center-of-mass (CM)
- r radius of orbit from CM
- n quantum level
- L angular momentum
- d total distance between charged masses orbiting each other
- $k = 1/4\pi\varepsilon_o$, ε_o is the permittivity of free space
- e charge of electron or proton

Subscripts:

- $_e$ electron
- $_p$ proton, Planck
- $_o$ orbital
- $_s$ spin
- $_t$ total
- $_n$ quantum level

Constants:

$c = 2.9979250 \times 10^8$ m/s = velocity of light

$\hbar$ = Planck's constant/2π = 1.05459×10^{-34} J-s

$m_e = 9.109558 \times 10^{-31}$ kg = mass of electron

$m_p = 1.672614 \times 10^{-27}$ kg = mass of proton

$\alpha = 1/137.0361$ = fine structure constant

$G = 6.67430 \times 10^{-11}$ N.m^2/kg^2

$r_e = 2.817940287 \times 10^{-15}$ m = electron radius