Application of SRT to Calculating Mercury's Perihelion Advance

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This article shows how SRT as applied to charged particles may be applied in an analogous manner to gravitational bodies. With such application and the creation of a computer program, the perihelion advance of Mercury is calculated. The results are compared with results from calculations based on General Relativity Theory. Important characteristics of gravity are illuminated, including gravity laws.

1. Introduction

Two earlier GED articles, by T.E. Phipps, Jr. [1} and M.E. Hassani [2], have inspired this author to work on the analogy between electromagnetic and gravitational forces. This author has shown in [3] how the concepts of SRT apply to interactions between relatively moving charged particles. Here he extends those concepts to gravitational interactions between relatively moving heavenly bodies.

Based on an analogy to the relationship between relatively moving charges, Phipps [1] developed formulas for velocity–dependent gravitational potentials. That article inspired this author to derive the velocity dependent gravitational force using SRT. This derivation begins by presenting a one-to-one correlation of the forces of relatively moving heavenly bodies to the forces between the relatively moving charges. This correlation depends on the hypothesis that a gravitational field exists about a massive body and has characteristics similar those of an electric field about a charged particle.

Linear four-dimensional Minkowski space-time is standard for SRT. This space-time is easier to visualize than the non-linear curved space-time of GRT. Understanding the complexities of formulating the math of GRT and of solving the resulting differential equations is not necessary to comprehend the present article. However, a formula derived from SRT is employed, and its derivation involves the complexity of four-vector math. This derivation is not presented in this paper, but is referenced [4].

The following analysis uses the mathematical sign convention that the attractive force between masses is positive, and that both masses as applied in the equations have positive values.

2. Analogies

The correspondence between the Newtonian law for gravitational force magnitude and the Coulomb Law for electric force magnitude is:

$$F = Gm_1 m_2 \big/ r^2 \Leftrightarrow kq_1 q_2 \big/ r^2 \quad , \tag{1}$$

where G is the gravitational constant, m_1 and m_2 are the attracting masses, and r is the distance between the masses, and where $k = 1/4\pi\varepsilon_0$, with ε_0 being the permittivity of free space, q_1 and q_2 are the charges of the interacting particles, and r is the distance between the particles.

The three dimensional electric field $\mathbf{e}_\mathrm{c}$ pervading the space about a charged particle is defined below, and is analogous to the three dimensional gravity field $\mathbf{e}_\mathrm{g}$:

$$\mathbf{e}_\mathrm{c} = kq\mathbf{r} \big/ r^3 \Leftrightarrow \mathbf{e}_\mathrm{g} = Gm\mathbf{r} \big/ r^3 \quad , \tag{2}$$

where $\mathbf{r}$ is a position vector extending from the charge or mass and r is its magnitude.

The forces $\mathbf{F}_\mathrm{e}, \mathbf{F}_\mathrm{g}$ are analogous, and defined as:

$$\mathbf{F_c} = q_1 \mathbf{e}_\mathrm{c} \Leftrightarrow \mathbf{F_g} = m_1 \mathbf{e}_\mathrm{g} \quad . \tag{3}$$

Application of the SRT version of the electric field of the moving charge as seen by a stationary charge is:

$$\mathbf{e}_\mathrm{c} = kq\mathbf{r} \Big/ \gamma^2 r^3 \left[1 - (v^2 / c^2)\sin^2\theta\right]^{3/2} \quad , \tag{4}$$

where v is the magnitude of $\mathbf{v}$, the relative velocity between the two charges, $\gamma = 1/\sqrt{1 - v^2/c^2}$, θ is the angle between $\mathbf{r}$ and $\mathbf{v}$. The constant $k = 1/4\pi\varepsilon_0$, and c is the speed of light.

Place the test charge q_1 at the stationary point, and you can create an expression that represents the total electrodynamics force between the stationary charge and the moving charge. This force consists of the electric Coulomb force and the magnetic force. This expression is good for relative velocity from zero up to c. This is why this author calls this formula "The Basic Electromagnetic Law". Relativists might say Eq. (4) determines only the electric field, and the magnetic field is something else. But the magnetic force emerges as the difference in the electric field of a stationary charge and the electric field of the moving charge as observed by a stationary charge or observer. The derivation of this SR formula can be found in books on Relativity such as Dr. Wolfgang Rindler's book [4].

If the hypothesis that the gravity field has characteristics similar to the electric field is correct, then the gravity field of a moving heavenly body as viewed from a stationary heavenly body (or stationary observer) may be expressed as:

$$\mathbf{e_{g2}} = G\gamma m_2 \mathbf{r} \Big/ \gamma^2 r^3 \left[1-(v^2/c^2)\sin^2\theta\right]^{3/2} \quad . \qquad (5)$$

The terms in (5) have the same meaning as the terms defined for (4), except G, gravitational constant, and m_2, the mass of the moving body, replace k and q_2, respectively. Multiplying (5) by m_1 creates a formula for the gravity force between relatively moving heavenly bodies with relative velocities from zero to c.

Eq. (5) is where the complete analogy between moving charges and moving masses breaks down. Notice the inclusion of the gamma factor, γ, in the numerator of (5). This expresses the relativistic mass increase of the moving mass m_2. Charges are invariant on being transformed from one inertial frame to another, and an isolated moving charge does not need a gamma factor.

A study of (5) reveals the intensity of the gravity field increases when the relative velocity vector $\mathbf{v}$ is at a right angle to the $\mathbf{r}$ vector. When the velocity vector $\mathbf{v}$ is in line or parallel to the $\mathbf{r}$ vector, the intensity of the gravity field is reduced depending on the magnitude of the velocity. This author makes no claim that this application of SRT theory replaces GRT, but does make the claim this force relationship is valid, and can be used to study relationships between relatively moving heavenly bodies. As evidence for the power of this application of SRT, this author has created a computer program that calculates the Mercury perihelion advance from a formula derived from (5). The program is also used for calculating the perihelion advance of Venus and Earth.

3. The Formula for Force Between Relatively Moving Heavenly Bodies ($v << c$)

Upon canceling gamma factors in the numerator and denominator of (5), and including the mass m_1 to create force per (3), and applying the binomial series to the factors in the resulting denominator, and eliminating higher orders of v^2/c^2, one arrives at the following formula:

$$\mathbf{F_{12}} = \frac{Gm_1 m_2 \mathbf{r}}{r^3}\left[1+\frac{v^2}{c^2}\left(1-\tfrac{3}{2}\cos^2\theta\right)\right] \quad . \qquad (6)$$

Eq. (6) is the formula that can replace Newton's Law for gravity for most applications. Note that this force law has a term dependent on speed v and direction angle θ, defined above for Eq. (4). This dependence on speed and direction is a consequence of SRT, and are included in the velocity dependent formula for gravity. Some other authors [1,2] who write about the velocity-dependent gravity field do not include the direction dependence.

Eq. (6) is analogous to the formula that represents the force between a stationary charge and a current element. That electrodynamics formula can be viewed in this author's paper [3].

4. A Computer Program for Testing the Gravity Law (6)

This author generated a Pascal program employing the speed and angle dependent term of (6) for computation on a standard IBM type PC. The intent was to calculate the perihelion advance of the planets Mercury, Venus, and Earth. This was satisfactorily achieved. While the program will not be presented in detail in this article, the following paragraphs will present the essential elements of the program. The program offers insight into how the perihelion advances of elliptical orbits occur. Double data type is employed in the program to provide 15-16 significant figures in the computations.

4.1 The Basic Approach of the Computer Program

A planet will traverse a perfect elliptical orbit about the Sun if only the Newton Gravity Force Law applies. This law varies in strength as $1/r^2$. But if the Relativistic Gravity Force Law applies, then the planetary orbit varies from a true elliptical orbit by a very small amount. For Mercury, the ratio of the added force caused by relativity to a true Newton force is 3.86×10^{-8} at perihelion. Therefore, the velocity of the planet used for the relativity term can be determined from the true elliptical orbit with little error. Also, the small planetary drift created by the relativity term may be modeled separately from the larger Newton Gravity Force.

An elliptical orbit is mathematically defined [5] for a given planet using the ellipse center reference origin and x and y coordinates. The ellipse semi-major axis, a_{smajor}, coincides with the x-axis and the semi-minor axis, b_{sminor}, coincides with the y-axis. The focus of the ellipse, the center of rotation of the Sun and Mercury, is placed on the positive x-axis at a distance equal to ε, the eccentricity. Perihelion is then on the positive x-axis at distance of a_{smajor} from the center of the ellipse. Inputs [6] to the program are the particulars of a given planet such as its mass, m_2; semi-major axis, a_{smajor}; eccentricity, ε; and speed at perihelion, v_{p}; and the time required for one complete orbit, T. Other inputs include the gravitational constant, G and the mass of the Sun, m_1, and the speed of light c.

The mass of the planet is modeled as moving for one orbit counterclockwise around the circumference of the rigidly defined orbit, starting at perihelion. Calculations are done with x values determined at regular intervals on the x-axis $(\Delta x = a_{\text{smajor}}/20)$. Computations are done for one-half of the orbit; then this same routine (for the first half of the orbit) is rotated 180 degrees and used again to calculate values for the second half of the orbit. Appropriate values generated in the first half of the orbit are carried over to the second half of the orbit. For the second half orbit, the focus is moved to the negative x-axis a distance of ε.

The total orbital path is divided into 80 spatial intervals corresponding to equal lengths Δx along the x axis, and 80 corresponding intervals of time, Δt with various numerical values computed by the program. Also computed at each cumulative x

value are the radial distance from the sun to the planet, the angle of $\mathbf{r}$ with respect to the x-axis, the angle of the planet's velocity vector with respect to the x-axis, the angle of the planet's velocity vector with respect to the radial vector $\mathbf{r}$, the angle between successive $\mathbf{r}$'s, and the speed of the planet.

The speed of the planet is calculated based on the difference in potential energy from one interval to the next, giving up that energy to a difference in kinetic energy. The initial potential energy and kinetic energies of the planet are calculated at perihelion.

The speed and angle-dependent force contribution from (6) is:

$$\Delta \mathbf{f}_{12} = \frac{G m_1 m_2 \mathbf{r}}{r^3} \frac{v^2}{c^2} \left(1 - \frac{3}{2} \cos^2 \theta \right) \quad . \tag{7}$$

This force implies a radial acceleration contribution of value:

$$\Delta a = \Delta f_{12} / m_2 = \frac{G m_1}{r^2} \frac{v^2}{c^2} \left(1 - \frac{3}{2} \cos^2 \theta \right) \quad . \tag{8}$$

The angle θ in (8) is the angle between the radial vector $\mathbf{r}$ and the velocity vector $\mathbf{v}$ of the planet's motion in its orbit. This angle is generally near $\pi / 2$, making the term involving θ small. For the planet Mercury, the factor in parentheses varies from a low of .937 to 1.0.

The program starts at perigee, where the velocity of the planet in the radial direction is zero. It numerically integrates (8) to find a velocity correction, and numerically integrates that to find a radial displacement correction.

The displacement increments from all 80 iterations for one complete orbit are summed and called s_{total} and d_{total}. These totals are at right angles with respect to each other. So they combine to:

$$s_{\text{hypotenuse}} = \sqrt{(s_{\text{total}})^2 + (d_{\text{total}})^2} \tag{9}$$

To calculate the perihelion advance one needs knowledge of how ds changes with respect to the angle change $d\theta$ of a radial vector $\mathbf{r}$. This is accomplished by noting that the circumference of a circle extends for an angle of 2π, so:

$$\Delta\theta = s_{\text{hypotenuse}} / r \quad . \tag{10}$$

A suitable r is obtained for the elliptical orbit by averaging the major and minor semi-axis radii: $r = (a_{\text{smajor}} + b_{\text{sminor}}) / 2$. The accumulated ds is then divided by this r at the end of the iterations. The perihelion advance is computed for 100 Earth years.

4.2 Computed Results for the Perihelion Advances of Three Planets

Table 1 compares results computed using the force formula (7) and the results computed using GRT. We see perihelion advances expressed in arc seconds per 100 Earth years for three planets.

Planets:	Computed Advance	GRT Calculated*	%Difference
Mercury	42.60	42.98	–0.89
Venus	8.18	8.62	–5.1
Earth	3.63	3.84	–5.5

*Data supplied by Wikipedia (internet): en.wikipedia.org/wki/Test_of_general_relativioty

The program for Mercury perihelion advance was run without the term containing θ in Eq. (7). The results were 43.92 arcsec per 100 Earth years, an increased difference of 2.18% compared to the GRT value.

5. Discussion of Results

The GRT calculated results presented in Table 1 are in agreement with observed results referenced to the ICRF, International Celestial Reference Frame. The computed advance results are relative to the Sun or more correctly relative to the center of rotation of the Sun and planet. As such they are essentially referenced to the ICRF. The computed results could not be in such close agreement with the observed results unless the force formula (7) is valid as a term of Eq. (6). The same computer program was used for calculating the perihelion for the three planets with only the input parameters changed. Eq. (7) causes the program to track the various perihelion advances of the three planets. The close agreements of the advances predicted by Eq. (7) strongly suggest that Eq. (6) is a valid refinement to Newton's gravity law.

The results unequivocally support the v^2 / c^2 term in (6). It is to be noted that the v^2 / c^2 term has a factor depending on the angle between the position vector $\mathbf{r}$ and the velocity vector $\mathbf{v}$. It represents the increase of the gravity field of the moving body when the velocity vector of the moving body is at or near perpendicular to the position vector between the two bodies. It shows a decrease in the gravity field of the moving body when it is moving away. The angle dependent term, having θ as a parameter, was derived from SRT and is 'locked' with the v^2 / c^2 term. The results for Mercury computed with the θ term show closer agreement with the GRT values than the results without that term. The difference is not large enough to form a definite conclusion, based on the computation alone, about the θ term.

Close agreement with the GRT calculated value was achieved with this computer model for the Mercury perihelion advance (-0.89% difference). This suggests that the computer model is a good one if the GRT value and observed value for Mercury are reliable. When the model is applied to the perihelion advances of Venus and Earth, there were larger error differences of 5.1% and 5.5% respectively. These error differences amount to an averaged difference of -0.34 arc sec/hundred Earth years for the computer model as compared to the GRT values.

The gravity law Eq. (6) could replace Newton's gravity law for some applications. It is strongly supported by the computed perihelion advances when compared with the observed advances. Here is a quote from Einstein's paper of 1915 when he discloses his application of GRT to the Mercury perihelion advance: "This calculation leads to the planet Mercury to move its perihelion forward by 43'' per century, while the astronomers give 45''±5'', an exceptional difference between observation and Newtonian theory. This has great significance as full agreement."[7]

As criticism to this SRT-based approach to gravity, here is a quote from Dr. Wolfgang Rindler, "Several attempts have been made to construct also new theories of gravitation within special relativity, but this can only be done at the heavy cost of abandoning the equivalence principle or a 'natural' interpretation of SR." [4] p. 76. He also said, referring to the linear approximations to GRT, "We end this chapter with a brief discussion of a subject that is important in many practical applications of GR, from gravitational waves to the physics of black holes: the linear approximation. This approximation to GR is usually much simpler to apply than GRT itself, though it must be applied with care; it sometimes gives results which in no way approximate to those of the full theory." [4] p. 188. This author admits to not having a full understanding of GRT. He speculates that GRT is a non-linear version of the linear SRT.

6. Expanding Universe

Eq. (5) extended to include the second mass is the more universal gravitational law:

$$\mathbf{F_{12}} = Gm_1 m_2 \mathbf{r} \Big/ \gamma r^3 \left[1 - (v^2 / c^2)\sin^2(\theta)\right]^{3/2} \quad . \qquad (11)$$

Eq. (11) is good for the relationship between two relatively moving masses with relative speed from 0 to c.

Observe from (11) that, if the direction of the relative velocity is in the direction of the $\mathbf{r}$ vector (moving away from each other), then the $\sin^2(\theta)$ term goes to zero and the equation is left with one γ factor in the denominator. Therefore, as the relative velocity increases to values large compared to c, the force of gravity attraction is considerably reduced, not only by large r, but also by large v. This formula applies to the expanding Universe, and is the more general of the two gravity formulas since (6) is restricted to relative speed much less than c. Most relative speeds encountered are much less than c.

Eq. (11) predicts that a spinning disc whose plane is in the vertical direction should weigh slightly less than when it is spinning at the same rate in a plane in the horizontal direction.

7. Conclusion

The perihelion advance of Mercury was one of the first 'proofs' utilized to support GRT. Now the same argument can be made with the same logic to support SRT. It is very significant that the perihelion advance of Mercury can be calculated from SRT. The use of Eq. (6) could simplify calculations that are complicated in GRT. SRT may model cosmic considerations when the distance is equal to or greater than the distance between the Sun and Mercury. That includes most of the universe. And space may be interpreted with four dimensions instead of 'curved space'.

These results suggest that the gravity force acts like a field force, similar to an electric force field. So a *gravity field* force is the probable cause of gravity. Thinking of gravity as a field versus as being 'curved space' is valid for the calculation of the Mercury Perihelion Advance. The gravity laws expressed as Eqs. (6) and (11) are supported with the results of the computer computations presented in this paper.

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