Mass Changes and GPS Atomic Clock Rate

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One of the purposes of this paper is to show how Special Relativity time dilation formula could have been interpreted wrongly by GPS scientist. And yet they have arrived, with the mis-interpretation, at the correct formulas for correcting the clock rate for the atomic clocks sent up in the GPS satellites. Also, this paper shows how scientists across the spectrum may have mis-interpreted time dilation in Special Relativity. Another purpose of this paper is to show how clock rate varies with mass change; an idea put forth by Viraj Fernando*. Another purpose is to show how centrifugal acceleration changes clock rates just as gravity does.

Introduction

This paper summarizes some of the information that has come to my attention mainly via Viraj Fernando. He has made the remarkable observation that the clock rate of an atomic clock increases in the amount proportional to its mass increase (or internal energy increase) cause by going from a lower altitude to a higher altitude in a gravity field. This fact is indisputable. It was verified by the observed clock rate adjustments made to the GPS atomic clocks before they are sent via satellites in orbit about the earth. The atomic clock rates in the satellites as *referenced to the surface of the earth* become faster at the higher altitude. Also, Fernando has shown, using the concept the mass of the atomic clock increases by gaining height, this formula:

$$\frac{\text{mass increase}}{\text{starting out mass}} \rightarrow \Delta \text{m/m} \rightarrow \frac{GM}{c^2}\left(\frac{1}{r_e} - \frac{1}{r_o}\right) \tag{1}$$

and

$$\frac{\text{clock rate increase}}{\text{starting out clock rate}} = \frac{\Delta \text{m}}{\text{m}} \text{ (sec. per sec. units)} \tag{2}$$

Where G is the gravitational constant, M is the mass of the earth, c is the velocity of light, r_e is the radius of the earth, and r_o is the radius (from the center of the earth) of the orbit of the GPS clock. Also note that $\Delta m = \Delta E / c^2$ where ΔE is energy change. It is assumed that the mass of an object increases due to its gravitational potential energy as it increases in altitude from the earth surface.

The fact that clock rates change proportionally to mass changes offers the possibility of a new interpretation for GR effects and SR effects.

Problem

The cause of the clock rate decrease that occurs when the GPS clock takes on orbital velocity (v_o) was in dispute between Fernando and myself. Fernando predicted a clock rate decrease due to a mass decrease due to just centrifugal force (as I understood his theory). In contrast, I assert that there is a clock rate decrease due to the centrifugal force and a clock rate increase as *referenced from the surface of the earth* due to mass increase due to kinetic energy of the orbiting clock. These last two differences are expressed in the following formulas:

$$\text{Fernando} \rightarrow \text{mass decrease due to centrifugal force} = \frac{1}{2c^2} \text{m} v_o^{\,2} \tag{3}$$

Note that Eq. (3) would be the result of using the time dilation formula of special relativity.

Keele $\rightarrow$ mass decrease = mass decrease due to centrifugal force - mass increase due to orbital kinetic energy

$$= \frac{\mathrm{m}v_o^{\ 2}}{c^2} - \frac{1}{2c^2}\mathrm{m}v_o^{\ 2} = \frac{1}{2c^2}\mathrm{m}v_o^{\ 2} \qquad (4)$$

In both of the above cases the mass decrease is in the correct amount that is needed to correct for the observed slowed down GPS clock rate due to orbital velocity. While the results are the same there is a very significant difference in interpretation of the cause of the orbital mass decrease and thus clock rate decrease.

Mass decrease due to centrifugal acceleration of the orbiting clock

The author will now proceed to justify his interpretation of the magnitude and cause of the mass decrease due to centrifugal force of the orbiting clock. Starting off:

$$\text{gravaty force} \leftrightarrow \frac{GMm}{r^2} = \frac{mv_o^{\ 2}}{r} \leftrightarrow \text{centrifugal force} \qquad (5)$$

In going from a position of r = infinity to a position of a finite r in a gravity field the mass due to gravitational potential energy decreases in the amount of (multiplying both side of the above equation by r):

$$\text{mass decrease (gravity)} \leftrightarrow \frac{GMm}{rc^2} = \frac{mv_o^{\ 2}}{c^2} \leftrightarrow \text{mass decrease (centrifugal force)} \qquad (6)$$

This last equation is the most significant thing about the author's argument. It shows that the centrifugal force or acceleration acts like a gravity field and has the same effects on the mass. This is due to the Equivalence Principle of General Relativity. Its effects is huge and apparently has hither to has not been recognized in physics (at least by the author).

Since:

$$v_o^{\ 2} = \frac{GM}{r_o} \qquad (7)$$

for an orbiting mass much smaller than M, the formula for GPS clock rate decrease due to centrifugal force is:

$$\frac{GM}{r_o c^2} \qquad (8)$$

Mass increase due kinetic energy of orbiting clock

The mass increase of the orbiting clock is:

$$\frac{mv_o^{\ 2}}{2c^2} \qquad (9)$$

Substituting Eq. (7) into Eq. (9) and dividing by *m*, the clock rate increase (or time inflation as contrasted to time dilation of special relativity) is:

$$\frac{1}{2c^2}\frac{GM}{r_o} \qquad (10)$$

The author contends that a time inflation formula of special relativity should be used here instead of the conventional time dilation formula of special relativity. This new time inflation formula is:

$$T = \gamma T' = \left(1 + \frac{v^2}{2c^2}\right) T' \text{ for velocities much less than } c \qquad (11)$$

The clock rate increase of Eq. (11) when considered as kinetic energy will produce Eq. (10).

Consolidating clock rate changes

Mass increase due to altitude/*m* (1) – mass decrease due to centrifugal acceleration/*m*(8) + mass increase due to kinetic energy/*m*(10):

$$\frac{GM}{c^2}\left(\frac{1}{r_e} - \frac{1}{r_o}\right) - \frac{GM}{r_o c^2} + \frac{1}{2c^2}\frac{GM}{r_o} \qquad (12)$$

$$= \frac{GM}{c^2}\left(\frac{1}{r_e} - \frac{1.5}{r_o}\right) \qquad (13)$$

Eq. (13) is the formula which can be used to calculate the amount the clock rate of the GPS atomic clock in the GPS satellite, when on the surface of the earth, needs to be reduced for special and general relativity corrections. It produces the same answer as calculated by GPS scientists. If you insert numbers for the parameters of Eq. (13), multiply by the number of seconds in a day, and do the calculation, you get 38.85088 microseconds/day. Finally, a Sagnac error correction is calculated in the receiver to make the earth surface clock match the satellite clock. This correction is independent of the clock rate setting.

Some Implications

GPS atomic clock rate error correction is a good way to test some of the aspects of GR and SR. Also, it is a good way to challenge our conventional way of thinking about physics. The following are some of the implications of the above analysis:

1. The mass gain or mass decrease due to gravitational potential energy change is associated with the body masses rather than "mass changes of the gravity field".
2. "Time Inflation" occurs in SR as well as "time dilation". Has "time dilation" been interpreted wrongly by physicists?
3. There is a GR relativity effect, like a gravity field, due to centrifugal acceleration.
4. The relative mass change effects as seen by an observer is directly associated with clock rate changes as seen by the observer.

Can a mass change interpretation of GR and SR be formalized and is mass concept basic?

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Also, I want to acknowledge the late Tom Van Flandern for his web article “What the Global Positioning System Tells Us about Relativity”.