Experimental Support for Einsteinian Electrodynamics

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An Einsteinian Electrodynamics is developed using a transformation of Einstein's special relativity (SR). Laws and formulas are presented for forces, energy levels, and induction between relatively moving isolated charges, between a current element and an isolated charge, and between current elements. These laws and formulas are based on Coulomb's Law and SR, both of which have been basic in conventional physics for over 100 years. Experiments designed and performed by the author support an induction formula from Einsteinian Electrodynamics as well as the classical Ampère's Law derived from his own experiments in about 1822. Einsteinian Electrodynamics reveals neglected coherencies between SR and earlier formulations, such as the classical Ampère's Law and Newton's Third Law. Experimental findings are consistent with its mathematical expressions, and it provides novel insights in the field of electrodynamics, for central concepts such as that of 'magnetic field' and 'induction' and for phenomena such as energy distribution in current elements (as in arc welding), the physics of force between current elements, radio wave propagation, gravity fields and sub-atomic nuclear-type orbits.
Key Words: electrodynamics, special relativity (SR), Lorentz transform, electric field (e-field), relative moving charges, magnetic force, Coulomb force, magnetic field energy, steady-state, dynamic, induction.

1. Introduction

Many scientists in the past have worked in the field of electrodynamics. Some of the more known, in the 1800's, were Oersted, Ampère, Biot, Savart, Guass, Weber, Grassmann, Neumann, Lorentz, Faraday, and Maxwell. Well-known contributors in the 1900's include, Einstein, Moon, Spencer, Pappas, Assis, Graneau, Klyushin, among others. Much of the work in the field was done before the electron was discovered, before it was learned how current flowed in a wire, before the invention of vector analysis, before the invention of the computer, before the advent of an equation editor for a computer, and before Einstein's discovery of special relativity.

This paper acknowledges Mario J. Pinheiro for his work "A reformulation of mechanics and electrodynamics" [1]. This paper especially acknowledges the work of Dr. Peter Graneau in his book, "Ampère-Neumann Electrodynamics of Metals" [2], because he supports and uses the old classical Ampère's Law as derived from SR. The derivation of Einsteinian Electrodynamics from SR yields new insights into the general field of electrodynamics.

Relativists have determined an equation for the transform of the electric field of a relatively moving electric charge. This equation Eq. (1) [3], [4] expresses the electric field intensity at a stationary point, emanating from a relatively moving charge q_1 at a 3-vector distance of $\mathbf{r}_1$ (See Figure 1),

$$\mathbf{e_1} = \frac{kq_1\mathbf{r_1}}{\gamma^2 r^3\left[1\text{-}\left(v^2/c^2\right)\sin^2\theta\right]^{\frac{3}{2}}} \qquad (1)$$

where v = magnitude of **v**, the uniform relative velocity between q_1 and a coordinate system or a stationary point in the coordinate system, $\gamma = 1/\sqrt{1-v^2/c^2}$, θ = angle between $\mathbf{r_1}$ and **v**. $\hat{\mathbf{r}}_1$, when used, is the unit vector of $\mathbf{r}_1$ and r is it's magnitude. The constants are $k = 1/4\pi\varepsilon_0$ (ε_0 = permittivity of free space) and c = speed of light. In the coordinate system, $q_1$is determined to be moving at the origin while q_2 is stationary, a convention used throughout this paper. Eq. (1), when multiplied by q_2 (See Figure 2), a test charged placed at the stationary point, represents the total electrodynamics force between the stationary charge and the moving charge. This force consists of the electric Coulomb force and the magnetic force. This expression is good for relative velocity from zero up to c.

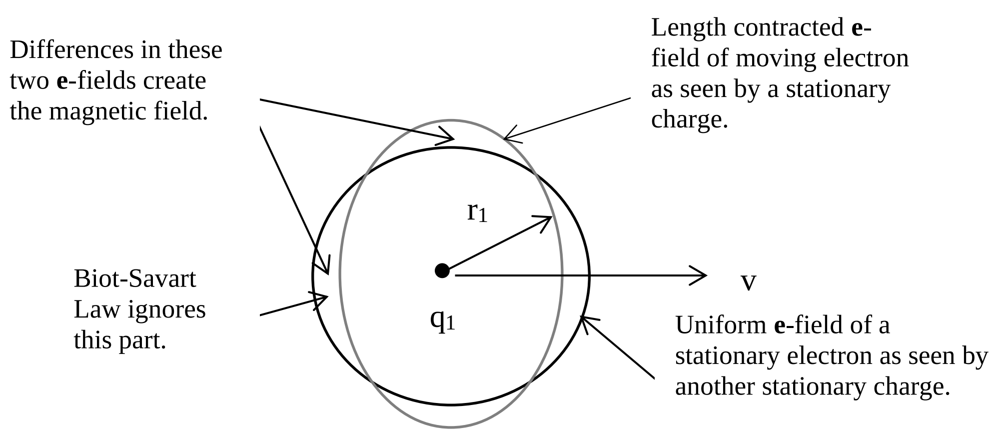

Figure 1. How special relativity (SR) creates the magnet field of moving charges. This shows a cross section of the effect on the e-field of a moving charge represented by Eq. (1).

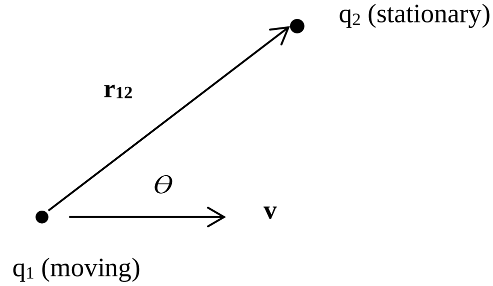

Figure 2. Relation between the vector terms in Eq. (1) and q_2.

Θ is angle between $\mathbf{r_{12}}$ and $\mathbf{v}$.

A problem with conventional electrodynamics begins with the Lorentz Force Law Eq. (2) and its inconsistency with a similar law derived from the application of SR. The Lorentz Force Law (three-force) is:

$$\mathbf{f} = q\left(\mathbf{e} + \mathbf{u} \times \mathbf{h}\right) \tag{2}$$

where **u** is a three-vector relative velocity between *q* and an object not clearly specified. According to Assis and Peixoto [5], Lorentz would say the object is absolute space and Einstein would say the object is the observer. In this paper the specified object is the magnetic field force requiring consideration of relative velocity of moving charges.

The second term in the equation is the vector cross product of **u** and **h**, **h** being the conventional magnetic field**.** Relativists, physicists such as Dr. Wolfgang Rindler [3], using four-vector math and SR's definition of four-force, use a Lorentz Transformation of the two terms of Eq. (2), one term that involves the **e**-field and the other term that involves the **h**-field, the conventional magnetic field. As a result Relativists derive Eq. (1) but also another equation for the **h**-field given as $\mathbf{h} = (1/c)v \times \mathbf{e}$. The Lorentz Force Law term for **h** should not be employed in the transformation when relating it to a moving charge and the resulting equation for **h** when it is used in the transformation should not be used. Confusion exists in this area with Dr. Rindler. Only the first term of the Lorentz Force Law, the **e**-field term, should be employed in the Four-force transformation. Then Eq. (1), when associated with a stationary test charge, will produce both the electric and "magnetic" forces between the relatively moving charges.

An equation very similar to the Lorentz Force Law is derived from Eq. (1) using the Binomial Series and eliminating higher order terms of v^2/c^2:

$$\mathbf{f_{12}} = q_2\left[\frac{kq_1}{r^2} + \frac{kq_1}{r_{12}{}^2}\frac{v^2}{c^2}\left(0.5\text{-}1.5\cos^2\theta\right)\right]\mathbf{\hat{r}_{12}} \tag{3}$$

Eq. (3) describes the total force between two relatively slow moving isolated charges. The first term describes an uniform **e**-field (Coulomb field) as in the Lorentz Force Law and the second term describes magnetic effects as does the second term in the Lorentz force law. Also, the relative velocity *v* between the two charges is clearly defined, as in Eqs. (1) and (3), and not obscured as in the Lorentz Force Law. The force in Eq. (3) acts along $\mathbf{r_{12}}$ and does not violate Newton's Third Law as the Lorentz Force Law does [6]. Eq. (3) could be written as:

$$\mathbf{f_{12}} = q_2 \left(e_1 + e_{m1} \right) \mathbf{\hat{r}_{12}} \tag{4}$$

where e_{m1} is the new definition of the magnetic field created by the moving charge q_1:

$$\mathbf{e_{m1}} = \frac{kq_1}{r_1^2} \frac{v^2}{c^2} \left(0.5 - 1.5\cos^2\theta \right) \mathbf{\hat{r}}_1 \tag{5}$$

The definition of a field, electric or magnetic, is force per unit charge. Consequently, multiplying e_{m1} by q_2 gives magnetic force between relatively slow moving isolated charges.

In their book "Electromagnetism" [7] Pollack and Stump applied special relativity to electrodynamics and applied the Lorentz Transformation just as Rindler had done, as described above. Pollack and Stump concluded that special relativity supports the Lorentz Force Law Eq.(2). But it is all based on using the second term in the Lorentz Force Law in their transformation. And it is illogical to use a formula at the start of the transformation to arrive at the same formula at the conclusion of the transformation.

Several electrodynamics laws emerge from the transformation shown by Eq. (1): the law for the magnetic force between slow relatively moving charges as described above, the law of magnetic force between a stationary charge and a current element, similar to Gauss's Law, and the classical law for magnetic force between current elements as determined by Ampère. These magnetic force laws are *steady-state* laws meaning the relative velocity between the charges is constant. Other formulas, included in this paper, describe *dynamic* forces, referring to those that fluctuate with time and are expressed as variables, and voltage induction formulas similar to Neumann induction formulas. Dynamic forces and induction forces are closely related.

2. The Magnetic Force Law between Slow Relative Moving Isolated Charges

2.1 The steady-state magnetic force law between slow relative moving isolated charges

This magnetic force law is derived from Eq. (1) by using the Binomial Series, eliminating the higher order terms of v^2/c^2 and subtracting out the stationary Coulomb force:

$$\mathbf{f_{12}} = \frac{kq_1q_2}{r_{12}^{\ 2}} \frac{v^2}{c^2} \mathbf{\hat{r}_{12}} \left(0.5\text{-}1.5\cos^2\theta \right) \tag{6}$$

This formula looks familiar. It shows the angle dependence of the magnetic force. This magnetic force law obeys Newton's Third Law and depends on the relative velocity. The force is in the direction of a line between the two charges (no torque is generated), and it does not depend on an ether. The charges are algebraic in the sense that a negative charge has a minus sign (-) associated with it and a positive charge has a plus sign (+) associated with it. A positive outcome of the equation would represent an increase in the repulsion of the two charges, and a negative result would represent attraction or a decrease in repulsion. Remember in this equation and all the following equations that θ is the angle between the *relative velocity vector* **v** and the radial vector $\mathbf{r_{12}}$ joining the two charges. Eq. (6) is a “steady-state” equation and describes the “magnetic force” only. In its derivation, the term for the “electric field force” has been subtracted.

3. The Magnetic Force Law between a Stationary Current Element and a Stationary Charge

3.1 The steady-state magnetic force law between a stationary current element and a stationary charge

This law Eq. (7) is derived from Eq. (1) in the same manner as Eq. (6) with the exception that q_1 in Eq. (1) is replaced with $\gamma\sigma_1 ds_1$, where σ_1 is line charge density and $\sigma_1 ds_1$ is the current element charge:

$$d\mathbf{f_{12}} = \frac{kq_2\sigma_1 ds_1}{r^2}\frac{v^2}{c^2}\hat{\mathbf{r}}_{12}\left(1\text{-}1.5\cos^2\theta\right) \tag{7}$$

The γ in $\gamma\sigma_1 ds_1$ expresses length contraction of the distance between the moving charges in the current element as seen by the test charge q_2 thereby increasing the effective charge density it sees. Notice the subtle differences between Eq. (6) and Eq. (7). The “0.5” in Eq. (6) is replaced by “1.0” in Eq. (7) and the moving charge q_1 is replaced by the current element charge $\sigma_1 ds_1$. Also, notice that Eq. (7) describes a force between a stationary charge and a “magnetic field” e_{m1} of a stationary current element. There is no need to have q_2 moving with respect to the current element as would be required by the Lorentz Force Law Eq. (2). This force has been reported in the works of Cooper [8] and Spencer [9].

Eq. (7) is very similar to Gauss’s Law (with the Coulomb term removed) derived in about 1835 [9]. Gauss’s Law is:

$$\mathbf{f_{12}} = \frac{kq_1 q_2}{{r_{12}}^2}\frac{v^2}{c^2}\hat{\mathbf{r}}_{\mathbf{12}}\left(1\text{-}1.5\cos^2\theta\right) \tag{8}$$

One or both of the charges in Gauss’s Law should be a current element charge Apparently Gauss, a mathematician, “back engineered” the classical Ampère’s Law to arrive at Eq.(8). The electron and special relativity had not been discovered when he derived this law. Gauss’s Law can denotes a force between relatively moving isolated charges, which differs from Eq. (6), the correct formula for the magnetic force between relatively moving isolated charges. It is important to recognize that Eq. (7) and Eq. (8) describes the magnetic force between a stationary charge q_2 or a stationary $\sigma_2 ds_2$, and a current element charge $\sigma_1 ds_1$. In Eq. (7) the velocity term v is the relative velocity between the positive ion lattice, which is stationary relative to q_2, and the moving electron lattice creating the current. When the equation is applied to the moving electron lattice of another current element the velocity term is the relative velocity of the electrons in that lattice and the moving electron lattice of the other current element.

Eq. (7) is employed to derive the classical Ampère’s Law [10]. Cross combination force relationships of Eq. (7), three altogether, are added together to form the classical Ampère’s Law as describes below. This is not a trivial derivation and is achieved here for the general 3-dimensional case using vector analysis. Once Ampère’s Law has been derived, the connection of special relativity to electrodynamics is confirmed. Peter Graneau’s work on “The Ampère-Neumann Electrodynamics of Metals” is then applicable [2]. Section 7, below, presents original experiments that support the classical Ampère’s Law and the related Neumann induction.

3.2 The dynamic magnetic force formula between a current element and a stationary charge

The following dynamic formula recognizes the time t_r for a change in the relative velocity to effect a change in the force between the current element and a stationary charge, and it depends on the speed of light. This time is assumed to be:

$$t_r = \frac{r}{c} \tag{9}$$

Differentiating Eq. (7) with respect to time, having r and θ constant, applying t_r, and recognizing acceleration $a = dv/dt$ we get:

$$d(d\mathbf{f_{12}}) = \frac{d(d\mathbf{f_{12}})t_r}{dt} = \frac{kq_2\sigma_1 ds_1}{r_{12}^{\ 2}}\frac{2va(r/c)}{c^2}\hat{\mathbf{r}}_{\mathbf{12}}\left(1\text{-}1.5\cos^2\theta\right) \tag{10}$$

Adding the steady-state to the differential we have:

$$d\mathbf{F}_{12} = d\left(d\mathbf{f}_{12}\right) + d\mathbf{f_{12}} = \frac{kq_2\sigma ds_1}{r_{12}^{\ 2}}\frac{v^2}{c^2}\hat{\mathbf{r}}_{\mathbf{12}}\left(1\text{-}1.5\cos^2\theta\right)\left(1+\frac{2ra}{vc}\right) \tag{11}$$

The dynamic part of this formula will be compared below (Section 10.4) to a formula for a radiating magnet field from a short wire transmitting antenna.

3.3 Induction

No new Einsteinian Electrodynamics laws need to be created to show how induction occurs. It is known from experience with induction that it can occur in a wire two different ways. One way (induction by movement) is to move a charge, wire or current element in a magnetic field, so that a voltage (emf) is acquired by the charge or is induced along the wire or current element. The other way (induction by changing magnetic field) is to vary the magnetic field created by a current in a wire or current element so that a voltage or emf is induced in a separated charge or current element. Since the magnetic field in Einsteinian Electrodynamics is a compressed or reduced **e**-field, Eq. (4), it is easy to determine the induced voltages using the existing Einsteinian Electrodynamics laws.

It is helpful to recall the definition of voltage from a physics book: “It is the work W done by a unit charge in passing between two points of a circuit equal to the *potential drop* between these two points. If W is now taken to represent the work done by the charge Q in moving between two such points, the potential drop between the points is $V = W/Q$. The term *potential difference* applies to both emf and potential drop; the practical unit is the volt. *The potential difference between two points is one volt if a charge of one coulomb either requires or expends one joule of energy in moving from one point to the other*” [11].

3.4 Induction by movement of a charge with respect to a stationary current element

The formula to be derived will be an elemental type formula as contrasted to formulas applied to macro circuits like large coils. Let $\mathbf{u_2}$ be the vector velocity of q_2 and $\mathbf{u_1}$ be the vector velocity of the electron lattice in the current element. Then $\mathbf{v} = \mathbf{u}_2 - \mathbf{u}_1$ and v is the magnitude of this relative velocity as used in the formula. β is the angle between $\mathbf{v}$ and $\mathbf{r_{12}}$. So the induced voltage will be:

$$dV_{emf} = df_{\mathbf{12}}dr = \frac{kq_2 q_1 \sigma_1 ds_1}{r_{12}{}^2}\frac{v^2}{c^2}\left(1\text{-}1.5\cos^2\beta\right)u_2\left(\hat{\mathbf{u}}_2 \bullet \hat{\mathbf{r}}_{12}\right)dt \tag{12}$$

where $v^2 = u_1{}^2 + u_2{}^2 - 2u_1u_2\left(\hat{\mathbf{u}}_1 \bullet \hat{\mathbf{u}}_2\right)$.

3.5 Induction on a stationary charge with respect to a stationary current element by varying current in the current element

This is the situation where a changing magnetic field induces a voltage on the stationary charge with the distance between the current element and the charge remaining constant. The energy stored in the field between the current element and charge is given by:

$$dE_{12} = \int_{\infty}^{r} \mathbf{f_{12}}dr \tag{13}$$

So that:

$$dE_{12} = -\frac{kq_2\sigma_1 ds_1}{r_{12}}\frac{v^2}{c^2}\left(1\text{-}1.5\cos^2\theta\right) \tag{14}$$

A change in dE_{12} due to a change in relative velocity would represent the work done on q_2. So that the induced emf voltage dV_{emf} is:

$$dV_{emf} = d(dE_{12}) = \frac{kq_2\sigma_1 ds_1 2v_1}{c^2 r_{12}}\left(1\text{-}1.5\cos^2\theta\right)dv_1 \tag{15}$$

Since $I_1 = \sigma_1 v_1$ then $dv_1 = dI_1/\sigma_1$. Substituting in Eq. (15):

$$dV_{emf} = d(dE_{12}) = \frac{kq_2 I_1 ds_1}{c^2 r_{12}}\frac{2}{\sigma_1}\left(1\text{-}1.5\cos^2\theta\right)dI_1 \tag{16}$$

4. The Magnetic Force Law between Current Elements

4.1 The steady-state magnetic force law between stationary current elements (classical Ampère's Law)

The test charge q_2 in Eq. (7) may be replaced by $\sigma_2 ds_2$ of another wire current element with charge line density σ_2. Then the two σds's are replaced by two Ids/v's, the v corresponding to the moving charges in each of the two current elements. The positive ion lattices of the two stationary current elements are stationary with respect to each other. Their Coulomb repulsion has already been subtracted out of Eq. (7) and this force does not need to be added. Then applying Eq. (7) three times to the cross combination of charges in the two current elements and adding the forces one arrives at the classical Ampère's Law [10] [12]:

$$d^2\mathbf{f_{12}} = -\hat{\mathbf{r}}_{12}\frac{kI_1I_2ds_1ds_2}{c^2{r_{12}}^2}\left(2\sin\alpha_1\sin\alpha_2\cos\eta\text{-}\cos\alpha_1\cos\alpha_2\right) \tag{17}$$

where $\hat{\mathbf{r}}_{\mathbf{12}}$ = unit vector in the direction of $\mathbf{r_{12}}$ and r_{12} = magnitude of the vector $\mathbf{r_{12}}$ joining the two current elements. The constants are $k = 1/4\pi\varepsilon_0$ (ε_0 = permittivity of free space) and c = speed of light. The I_1 and I_2 are current magnitudes and ds_1 and ds_2 are current element lengths. The angles are: α_1 = angle between ds_1 and $\mathbf{r_{12}}$; α_2 = angle between ds_2 and $\mathbf{r_{12}}$; η = angle between the plane of ds_2 with $\mathbf{r_{12}}$ and the plane of ds_{12} with $\mathbf{r_{12}}$.

Using vector analysis the classical Ampère's Law is mathematically equivalent to and is more conveniently expressed as:

$$d^2\mathbf{f_{12}} = \text{-}\hat{\mathbf{r}}_{12}\frac{kI_1I_2ds_1ds_2}{c^2{r_{12}}^2}\left(2d\hat{\mathbf{s}}_1\bullet d\hat{\mathbf{s}}_2\text{-}3(d\hat{\mathbf{s}}_1\bullet\hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2\bullet\hat{\mathbf{r}}_{12})\right) \tag{18}$$

where $d^2\mathbf{f_{12}}$ is the force on current element 1 caused by current element 2, $\mathbf{r_{12}}$ is the vector displacement from 1 to 2, and I_1 and I_2 are current magnitudes in current elements 1 and 2 respectively. The constants are $k = 1/4\pi\varepsilon_0$ (ε_0 = permittivity of free space) and c = speed of light.

A study of Ampère's Law reveals that successive current elements with current in the same direction repel each other. This fact was illustrated by Peter Graneau's experiments [2] of exploding wires by conducting huge currents through them. Parallel current elements with current in the same direction attract each other.

Eq. (18) is an instantaneous action at a distance formula involving constant currents in the current elements. It does not have a term that involves retardation of the field between the current elements. It is postulated the electric field of a moving charge with constant velocity has already pervaded all the space of its inertial frame. When acceleration affects the relative velocity, the magnetic force is affected by the change in velocity, but at a retarded time until the change in the electric field arrives at speed c, in most cases, at the other current element. This effect is not expressed in the Einsteinian Electrodynamics Eqs. (6), (7), (17), (18), and (38). Where distances are short, this effect is negligible. Weber in about 1845 wrote and published an equation identical to Eq. (6), with the exception that he added an acceleration term [9] (see Section 6.3).

The angle dependencies of Eq. (17) and Eq. (18) were tested and validated in the paper "Experiment with Ampère's Law and the Current Element" [13].

4.3 Induction between a stationary current element and a moving current element

Let $\mathbf{u}_1$ be the velocity vector of $d\mathbf{s}_1$ with respect to $d\mathbf{s}_2$, then:

$$d^2V_{emf}(t) = (d^2 f_{12})\frac{dr_{12}}{dt} = \text{-}\frac{kI_2ds_2I_1ds_1}{c^2{r_{12}}^2}\left[2d\hat{\mathbf{s}}_1\bullet d\hat{\mathbf{s}}_2\text{-}3(d\hat{\mathbf{s}}_1\bullet\hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2\bullet\hat{\mathbf{r}}_{12})\right]\left(\mathbf{u}_1\bullet\hat{\mathbf{r}}_{12}\right) \tag{19}$$

4.4 Induction between two stationary current elements with varying current in one element

In this case, we integrate Eq. (18) from ∞ to r to get the energy in the field between the two current elements and then vary the current in one of the current element to vary its field energy. The change in field energy then induces a voltage in the other current element. These current elements can be in the same circuit or each in separate circuits:

$$d^2V_{emf}(t) = \frac{d(d^2E_{12})}{dt} = \frac{kds_1ds_2I_2}{c^2r_{12}}\frac{dI_1}{dt}\left[2d\hat{\mathbf{s}}_1 \bullet d\hat{\mathbf{s}}_2 \text{-}3(d\hat{\mathbf{s}}_1 \bullet \hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2 \bullet \hat{\mathbf{r}}_{12})\right] \tag{20}$$

This formula is tested below in an experiment with current elements in separate circuits. It expresses a voltage induced in current element ds_2 caused by a varying current in current element ds_1 when I_2 is set to 1amp. Also, all terms not including the current terms on the right side of Eq. (20) and the minus sign, represent the mutual inductance between the two current elements:

$$d^2L_{M12} = d^2L_{12} = \frac{kds_1ds_2}{c^2r_{12}}\left[2d\hat{\mathbf{s}}_1 \bullet d\hat{\mathbf{s}}_2 \text{-}3(d\hat{\mathbf{s}}_1 \bullet \hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2 \bullet \hat{\mathbf{r}}_{12})\right] \tag{21}$$

The following Eq. (22) is also equal to Eq. (21) replacing k with μ_o, the permeability of free space:

$$d^2L_{12}^M = \frac{\mu_o ds_1ds_2}{4\pi r_{12}}\left[2d\hat{\mathbf{s}}_1 \bullet d\hat{\mathbf{s}}_2 \text{-}3(d\hat{\mathbf{s}}_1 \bullet \hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2 \bullet \hat{\mathbf{r}}_{12})\right] \tag{22}$$

L_{M12} for two separate circuits can be calculated on a computer using finite current element size:

$$L_M = \sum_1^{n_1}\sum_1^{n_2} d^2L_{12} \tag{23}$$

So the induced voltage based on Eq. (20) and dropping the minus sign for open circuit 2 is:

$$V_M = L_M \frac{dI_1}{dt} \tag{24}$$

With $V_1 = V_{1\max}\sin(wt)$, $I_1 = -I_{1\max}\cos\left(wt\right)$, $\frac{dI_1}{dt} = I_{1\max}w\sin\left(wt\right)$, and $I_{1\max} = \frac{V_{1\max}}{wL_{S1}}$ (25)

$$V_M = V_{1\max}\frac{L_{M12}}{L_{S1}}\sin(\omega t) \tag{26}$$

An experiment, presented below, is performed for varying conditions of L_{M12} to compare the calculated V_M with the measured V_{2emf}. Eq. (22) and (23) are employed to calculate L_{M12}.

4.5 Self Inductance

It is shown next how to calculate the self-inductance L_1^S of a single circuit. It involves just changing the limits of the summations of Eq. (23):

$$L_S = \sum_1^{n-1}\sum_{n_1+1}^{n} 2d^2L_{12} \tag{27}$$

Eq. (27) computes one-half of the self-inductance L_S. The factor of 2 in Eq. (27) is based on the formula for energy in a single coil: $E = I^2L/2$.

4.6 An example of induction is the self-induction law

$$V = L\frac{dI}{dt} \tag{31}$$

where V is the back electromotive force (emf) voltage induced in a coil due to a changing current dI/dt and the coil's inductance L. The value of the inductance L of a simple coil is calculated by the classical Ampère's Law which is derived from SR [10]. A popular formula for calculating the voltage induced in a coil is:

$$V = N \times d\varphi / dt \tag{32}$$

where V is the voltage induced in a coil of N turns due to a changing magnetic flux φ with time t. Eq. (32) involves the concept of a magnetic field (magnetic flux density) B employed by Faraday, Lorentz, and Maxwell. This concept of the magnetic field is a mathematical aid to handle macroscopic circuits that involve electrical currents. It works well in engineering applications and some physics applications. This concept of the magnetic field summarizes gross conditions in electrical circuits, but it fails at the elemental level. (Maxwell applied Faraday's gross results to guess at his equations for electromagnetic wave propagation).

5. A Prior Magnetic Law

5.1 Biot-Savart Law

The Biot-Savart Law for the current element is:

$$d\mathbf{B} = kI\,\frac{(d\mathbf{s} \times \mathbf{r})}{r^3} \tag{33}$$

where $\boldsymbol{B}$ is the magnetic flux vector, I is the scalar current in the current element length of vector $d\boldsymbol{s}$, and $\boldsymbol{r}$ is the vector distance from the field to the current element. This law, supposedly, works at the macro level when integrated around a loop where the real magnetic forces between adjacent current elements cancel out, but it fails at the current element size because it does not recognize the magnetic forces between adjacent current elements. The classical Ampère's Law, derived from experiment and SR, does not fail in this regard. Also, this classical law gives insight into how a magnetic field is created. The energy of a magnetic field is related to the force between *two* current elements or *two* relative moving charges.

The concept of needing *two* objects to create electric and magnetic fields is important. For instance, the creation of an electron and a positron creates the electric field of the electron or the positron. The field of an isolated electron goes off into nether land, but somewhere its electric field terminates on a positive charge or charges. Likewise, it takes *two* objects to create a magnetic field. The SR magnetic field may be defined as the differential relativistic electric field created by a moving charge as it moves relative to a stationary charge, thus involving *two* objects. This concept might give clues as to the composition of a photon. It can be said the **e-**field of a photon is created by two massless charges that are alternately created and annihilated from space. As they are annihilated by moving toward each other, they create the relativistic magnetic field. So the photon alternates between the electric and magnetic field energies. This concept can help explain how polarization exists in photons of some electromagnetic radio waves.

Other magnetic force laws between current elements are discussed briefly below.

6. Some Existing Magnetic Force Laws

6.1 Lorentz/Grassmann's Law

Lorentz/Grassmann's Law is the combination of part of the Lorentz force law and the Biot-Sarvart Law [6]. The part of the Lorentz Force Law in differential form is:

$$d\mathbf{f_{12}} = I_1 d\mathbf{s_1} \times d\mathbf{B_2} \tag{34}$$

The Biot-Sarvart Law in differential form is:

$$d\mathbf{B_2} = \frac{\mu_o I_2 ds_2}{4\pi r_{12}^{\ 2}}\left(d\hat{\mathbf{s}}_\mathbf{2} \times \hat{\mathbf{r}}_\mathbf{12}\right) \tag{35}$$

Combining Eqs. (34) and (35) and applying the triple vector product identity results in the Lorentz/Grassmann's Law:

$$d^2\mathbf{f_{12}} = \frac{\mu_o I_1 I_2 ds_1 ds_2}{4\pi r_{12}^{\ 2}}\left[\left(d\hat{\mathbf{s}}_\mathbf{1} \cdot \hat{\mathbf{r}}_\mathbf{12}\right) d\hat{\mathbf{s}}_\mathbf{2} - \left(d\hat{\mathbf{s}}_\mathbf{1} \cdot d\hat{\mathbf{s}}_\mathbf{2}\right)\hat{\mathbf{r}}_\mathbf{12}\right] \tag{36}$$

Eq. (36) is the currently used *classical force formula,* employed in current textbooks. Pappas [6] discussed the merits of this Lorentz/Grassmann's Law as compared with the Ampère's Law Eq. (18). He stated that Ampère's Law is, in general, not relativistic or Lorentz invariant and violates energy conversation. He says the Lorentz law is relativistic or invariant for isolated charges. He also says that for closed circuits with uniform charge mobilities, both laws' deficiencies vanish, and both laws become identical. His assessments conflict with the assessments shown in this paper: The Ampère's Law is shown in this paper to have been derived from special relativity. And it obeys Newton Third Law. Energy conservation is not an issue. One can see from the Lorentz/Grassmann's Law Eq. (36) that the $d\mathbf{s_2}$ vector is not necessarily in line with $\mathbf{r_{12}}$, the vector joining the two current elements. Therefore, this law violates Newton's Third Law. This fact alone severely discredits Lorentz/Grassmann's Law as physically and mathematically unacceptable whereas Ampère's Law does not violate Newton's Third Law. The Lorentz/Grassmann's Law is non-derivable from special relativity. One can see from Eq. (36) that the force between inline current elements vanishes, but to be able to calculate any energy in a magnetic field one has to assume the first term on the right vanishes when integrated around a closed loop and the second term on the right is left to calculate the energy of the magnet fields with inline elements without any force between them. This makes it very difficult for student physicists who understands math to learn electromagnetism in some of the current popular college textbooks [7], [14].

Because of its popularity, inductance "calculated" using Eq. (36) is compared in experiments, where the inductance of several coils are measured, later reported in this paper (see Section 7.6 and Section 7.7, Note 6). The calculations are based on imperfect math, but best as can be applied to Eq. (36).

6.2 New Gaussian force equation

Domina Eberle Spencer and associates [9] developed an equation to express the force between relatively moving charges to replace Gauss Law Eq. (8). When their equation is applied to current elements the resulting equation is:

$$d^2\mathbf{F_G} = \frac{1}{8\pi\varepsilon_o c^2}\left[\begin{array}{l}\frac{I_1 I_2}{r^2}\left(\left(d\mathbf{s_1}\cdot\mathbf{a_r}\right)d\mathbf{s_2} + \left(d\mathbf{s_2}\cdot\mathbf{a_r}\right)d\mathbf{s_1} - 2\left(d\mathbf{s_1}\cdot d\mathbf{s_2}\right)\mathbf{a_r}\right) \\ + \frac{\left(\frac{dI_1}{dt}\right)I_2}{cr}\left(\left(d\mathbf{s_2}\cdot\mathbf{a_r}\right)d\mathbf{s_1} - \left(d\mathbf{s_1}\cdot d\mathbf{s_2}\right)\mathbf{a_r}\right)\end{array}\right] \tag{37}$$

where $\mathbf{a_r}$ is a unit vector in the direction of the vector connecting the two current elements. Eq. (37) is the New Gaussian Equation after triple vector product relations have been applied to the cross product terms in their original equation. Spencer states this equation satisfies many experiments and is most viable of all the magnetic force equations for current elements. She also states that it satisfies the repulsion attributes of successive current elements as observed by Peter Graneau. Doing all that, the equation would seem to deserve considerable merit. The author attempted to program this equation into the computer to calculate the inductance of a square coil. By noting that dI_1/dt can be set to zero eliminating the second major term of the equation, the first major term can be applied to calculate the inductance. When the inductance of just one side of the square coil was to be calculated, it was determined the equation produced zero inductance or zero energy storage in one side, thus having no force of repulsion between in-line current elements. This is contrary to Peter Graneau's experience [2]. When the inductance or energy storage was to be determined by current elements at right angles to each other, it was found that the vector established by the two current elements in the equation are not always in line with the radial vector $\mathbf{a_r}$, thus violating Newton's Third Law. It suffers the same defect as the Grassmann's Force Law in that respect. Spencer based the derivation of this equation on a couple of Gauss's Criteria [9]. In this reference, she also summarizes the development of the field of electrodynamics, making it an important paper. Spencer's main criticism of Ampère's Law, Eq. (18), invoked Gauss's second criterion for a law to be a law of electrodynamics: *The equation for force between moving charges must take into account the fact that electromagnetic effects are propagated at a very large but finite velocity.* Eq. (18) requires an infinite velocity. This is not a serious deterrent in the practical use of the equations of Einsteinian Electrodynamics since the distances involved are usually short, and when it is recognized that the equations represent "steady state" conditions where the relative velocity is constant or uniform.

Nor is this circumstance a serious deterrent in the practical uses of these equations since the distances involved in the use of the equations are usually short and changes in the fields due to current changes propagate at the speed of light. The approach to current changes or acceleration of moving charges was presented above as a dynamic equation. The section on radio wave propagation below will show some of the dynamics effects.

Spencer said this in her paper: "In the last chapter of his great book on electrodynamics Maxwell says that an equation for a force between moving charges is the 'keystone of electrodynamics'. But Maxwell never found such an equation that satisfied him". It is asserted Eq. (1) with q_2 inserted is the keystone equation:

$$\mathbf{F}_{12} = q_2\mathbf{e}_1 = \frac{kq_2q_1\mathbf{r}_1}{\gamma^2 r^3\left[1\text{-}\left(v^2/c^2\right)\sin^2\theta\right]^{\frac{3}{2}}} \tag{38}$$

Eq. (38) is a "steady state" equation having uniform relative velocity. It is good for constant, high relative velocity between the charges. It presents the total electric and magnetic force between moving charges.

6.3 Weber force equation

Andre Koch Torres Assis, a professor at the University of Campinas – UNICAMP, in Brazil has studied and worked for many years with Weber's unpublished equation [15], the left side of Eq. (39):

$$\mathbf{F}_{12} = \frac{q_1q_2}{4\pi\varepsilon_o}\frac{\hat{\mathbf{r}}}{r^2}\left(1-\frac{\dot{r}}{2c^2}+\frac{r\ddot{r}}{c^2}\right) = \frac{q_1q_2}{4\pi\varepsilon_o}\frac{\hat{\mathbf{r}}}{r^2}\left[1+\frac{1}{c^2}\left(\mathbf{v}_{12}\bullet\mathbf{v}_{12}-\frac{3}{2}\left(\hat{\mathbf{r}}\bullet\mathbf{v}_{12}\right)^2+\mathbf{r}_{12}\bullet\mathbf{a}_{12}\right)\right] \tag{39}$$

The left side of Eq. (39) can be converted to vector notation, as shown on the right side of Eq. (39). Eq. (39) can further be converted to:

$$\mathbf{F}_{12} = \frac{q_1q_2}{4\pi\varepsilon_o}\frac{\hat{\mathbf{r}}}{r^2}\left[1+\frac{v_{12}^{\ 2}}{c^2}\left(1-\frac{3}{2}\cos^2\theta\right)+\frac{ra_{12}}{c^2}\cos\phi\right] \tag{40}$$

where θ is the angle between the relative velocity $\mathbf{v}_{12}$ and $\mathbf{r}_{12}$, and ϕ is the angle between the relative acceleration $\mathbf{a}_{12}$ and $\mathbf{r}_{12}$. According to Assis the acceleration term on the far right of Eq. (40) represents force due to induction, but that aspect of this equation is not pursued in this paper. For uniform relative velocity the acceleration term drops out and the equation is left in the magnetic part of the equation that agrees with Gauss's Law and with the Einsteinian Eq. (7). So Gauss's Law and Weber's Law can match the Einsteinian Eq. (7) if at least one of the charges is a current element. Thus both Gauss's Law and Weber's Law, just as Einsteinian Eq. (7), can be used to derive the classical Ampère's Law Eq. (18). This is remarkable fact and it was not promoted by Maxwell and Lorentz.

Assis developed what he terms "relational mechanics" using Weber's equation. He uses the term "relational" instead of "relative" to distinguish his work from SR. These terms are used in the same way. He does not agree With SR. He applies Weber's equation to gravity much the same as this author has applied Eq. (1) to gravity (see Section 10.6).

6.4 Maxwell-Whittaker force equation

This equation promoted by Maxwell and later taken by Whittaker [16] is:

$$d\mathbf{f}_{12}^2 = \frac{\mu_o I_1 I_2 ds_1 ds_2}{4\pi r_{12}^{\ 2}}\left[-\left(d\hat{\mathbf{s}}_1\cdot\hat{\mathbf{r}}_{12}\right)d\hat{\mathbf{s}}_2-\left(d\hat{\mathbf{s}}_2\cdot\hat{\mathbf{r}}_{12}\right)d\hat{\mathbf{s}}_1+\left(d\hat{\mathbf{s}}_1\cdot d\hat{\mathbf{s}}_2\right)\hat{\mathbf{r}}_{12}\right] \tag{41}$$

Eq. (41) is similar to the steady-state part of Spencer's Eq. (37) and some aspects of it are similar to the Lorentz/Grassmann's Law. It suffers the same deficiencies of those two laws as described in Section 6.1 and 6.2.

7. Experiment to Verify the Classical Ampère's Law

Since the classical Ampère's Law can be derived using SR, an experiment to check on the validity of the classical Ampère's Law can be performed. Using a computer to calculate the energy between current elements is most easily done by selecting a length for the current element where the current element has canceling stored energy in it. Thus the stored energy in the current element does not have to be added to the rest of the energy stored in the combinations of current elements in a coil. The inductance of a one-turn square shaped coil is both calculated by the integral of Ampère's Law and measured by resonance with a calibrated capacitor.

By varying the length of the current element a match between the measured and calculated inductance can be obtained for a single turn coil. The inductance of the current element itself was calculated using the integral of Ampère's Law. The inductance of the length required for the match was found to be zero. There are mechanisms in the current element that give up energy to supply the energy stored in it so that the net stored energy is zero.

Eq. (22), Eq.(27) and Eq.(28) of Section 4.4 and Section (4.5) are programmed in a computer to calculate the self-inductance of a single turn coil, $L_1^S = L_c$. This suggests an experiment to test Ampère's Law where the inductance L_m of a single turn coil is measured by precise electrical techniques. One technique involves measuring the resonant frequency of the coil with known capacitor capacitance and calculating L_m. The total energy stored in the coil is given by:

$$E_t = \frac{I^2}{2} L_m \tag{42}$$

L_c and L_m can be compared directly, the current I through the coil being arbitrary. The inductance L of the coil is based on its physical characteristics. This makes for a great experiment.

7.1 Test Equipment

1) HP 3490A Multimeter (Used to calibrate resistor and capacitor values and to measure rms voltages)
2) HP 5360A Computing Counter with 5365A Input Module (Used to measure frequency of Test Oscillator)
3) HP 652A Test Oscillator (Used to supply sine wave voltage for achieving resonance in LC parallel circuit)
4) Tektronix 2245 100 MHz Oscilloscope (Used for detecting resonance of LC parallel circuit)

7.2 Single turn coil description

A square coil of 0.47 m/side was constructed (see Figure 3). A wooden multi-coil form was employed to support the corners of the square coil. Care was taken to eliminate most all material that could affect the value of inductance L. Insulated wire of four different diameters were tested. The diameters of the wires were: 0.203 mm, 0.635 mm, 1.024 mm and 2.053 mm. It is believed the insulation on the wire has no effect on the inductance of the coil.

Figure 3. Square coil form with 0.47 m to a side.

7.3 Capacitor Calibration

A calibrated resistor R was placed in series with the capacitor C to be calibrated and which was connected to ground. The test oscillator 3) at its 50 ohm output was connected to R. The ac RMS voltage v_c as measured across C to ground, and v_r across R to ground with Multimeter 1). The frequency f was measured with 2). Then the following formula was used in a computer to calculate C:

$$C = \frac{\sqrt{\left(\frac{v_r}{v_c}\right)^2 - 1}}{2\pi f R} \tag{43}$$

A mica capacitor was employed: 4700 pf +/- 1% (Calibrated to 4.733 x 10^{-9} farads)

Inductance Measurement L_m:

The following formula computes L_m:

$$L_m = \frac{1}{C\left(2\pi f_o\right)^2} \tag{44}$$

where f_o is the measured frequency at resonance and C is the calibrated value of the capacitor which is connected in parallel with the square coil. Power is supplied through a 1.5 k ohm resistor to the parallel circuit from the test oscillator 3).

7.4 Computing the Inductance L_c

A computer program using Eq. (22), Eq. (27), and Eq. (28) is employed to calculate the inductance L_c for each of the different wire diameters. The current element length is varied by changing the number of current elements to a side of the square coil. When the computed L_c value matches the measured value L_m, then the value of the length of the current element ds is noted and entered into the program that computes the energy or inductance in the current element.

7.5 Program for current element inductance

Since the inductance of the current element itself was not added in the program to calculate L_c, it is important to check if it was zero for the match made in computing L_c. In order to do this a program was written to view the current element as a group of very small parallel wires which carry current in the same direction. To formulate the wire group, the cross section area of the wire is determined from the coil's wire diameter and reshaped into a square cross section area of the same size. Then an integral number N dividing the side of the square cross section area is entered. N^2 represents the number of the small parallel wires. The target length of the current element is entered from the program calculating L_c. And a sub current element length is then determined for all the small parallel wires. Eq. (22), Eq. (27), and Eq. (28) are employed to calculate the inductance of the current element and to verify that it is zero or near zero since the program outputs discrete approximations. N is varied from 2 to 9 and the inductance or energy is found to change signs (meaning zero inductance) for all the wire diameters.

7.6 Results of the measurements and calculations

Table 1. Square Coil, 0.47m/side. Inductance Measurements and Calculations.

Solid wire diameter mm	ds required for match mm	Ratio: ds/wire diameter	L_m µH	L_c µH	ds in current element program	Number 'wires' in current element for zero energy[I]
0.203	1.661	8.182	2.914	2.915	1.661	49 to 64
0.635	5.165	8.134	2.488	2.484	5.165	36 to 49
1.024[II]	8.896	8.688	2.284	2.284	8.896	64 to 81
2.050	16.206	7.905	2.036	2.041	16.210	36 to 49

[I]Zero energy storage is used here to mean as well zero inductance, according to Eq. (42).
[II]Stranded.

7.7 Discussion of Results

1. Results support the classical Ampère's Law, Eqs. (17) and (18), and thus support the Einsteinian Electrodynamics Eqs. (1), (6), (7), (17), (18) and (38). This is because the classical Ampère's Law is derivable from Einsteinian Electrodynamics and supported here by appropriate and verifiable experiments.
2. Results show that inductance increases as wire diameter decreases even though the area of the coil remains the same.

3. Results show the length of the current element required for inductance match increases as wire diameter increases.
4. The average ratio of current element length used in the calculations to the round wire diameter is ds/w. dia. = 8.074 for solid wire. For the one stranded wire tested, this ratio was 8.7. This length creates around zero stored energy in the current element. In order to compute the inductance of a solid wire coil without the knowledge of its measured value, one would need to use this number (ds/w. dia. = 8.074) to approximate the current element length, ds, in this program. Precision is lost in this situation since this ratio varied a little in different size wires.
5. While Item 1 above, is correct, some physicists may claim the Current Element Program (see Section 9, Item 4) is not adequate to validate zero energy in the current element. It appears a current element of a length such that its inductance or stored energy is zero is a necessity for proper calculation of the inductance. With a ds/w. dia. ratio = 1, for instance, the program outputs an inductance that does not match the measured value.
6. Inductance of the single turn square coil of Table 1 was computed using just the right term of the right side of Eq. (36). Generally they matched the measured values L_m of Table 1 only by dramatically increasing the number of current elements to a side of the square coil. This reduced the ratio of ds/wire diameter to around 0.42. Of course with no in-line current element forces expressed by Eq. (36), there is no way to determine energy storage in this type of current element. It expresses a weakness of both Eq. (18) and Eq. (36) in that you can vary the current element size and change the value of the computed inductance. This places reliance on the zero energy concept of the current element for proper dimensioning of the current element size for use in Eq. (18).

8. Experiments with Induction

8.1 Experiment with induction with two square coils

Using the conditions described in the above Section 4.4 on induction between two stationary current elements with varying current in one element and Eq. (22) to (26), an experiment on induction in Einsteinian Electrodynamics was performed. A wooden multi-coil form was constructed much like four table legs around which five single turn coils were wound. (See Figures 4 & 5) The coils were square being 0.47 meters to a side. 18 a. w. g. stranded wire made up the coils. The first coil was the primary coil on which an oscillating voltage was applied. The other four coils were spaced parallel on the “legs” from the primary coil at 0.5 in., 1.0 in., 2.0 in., and 3.0 in. These four coils were left open circuited. The process of this experiment is as follows: Calculate with computer the mutual inductance of each of the four coils with respect to the primary coil that had the oscillating current. Then calculate the induced voltage on the coils. The calculated induced voltage was then compared with the measured induced voltage on the coils. Peak-to-peak voltages were measured on both the primary coil and all the secondary coils with the oscilloscope used in the above experiment. Also, the RMS voltages were measured with the multi-meter listed in the above experiment. A 4733 pf capacitor was placed in parallel with the primary coil and the output of the test oscillator was connected directly to the parallel circuit of the primary coil and capacitor and the frequency adjusted to cause resonance. The primary voltage was set to 8.0 volts p-p. Table 2 presents the results of the experiment with multi-meter measurements (scaled to having 8.0 v p-p on the primary coil.)

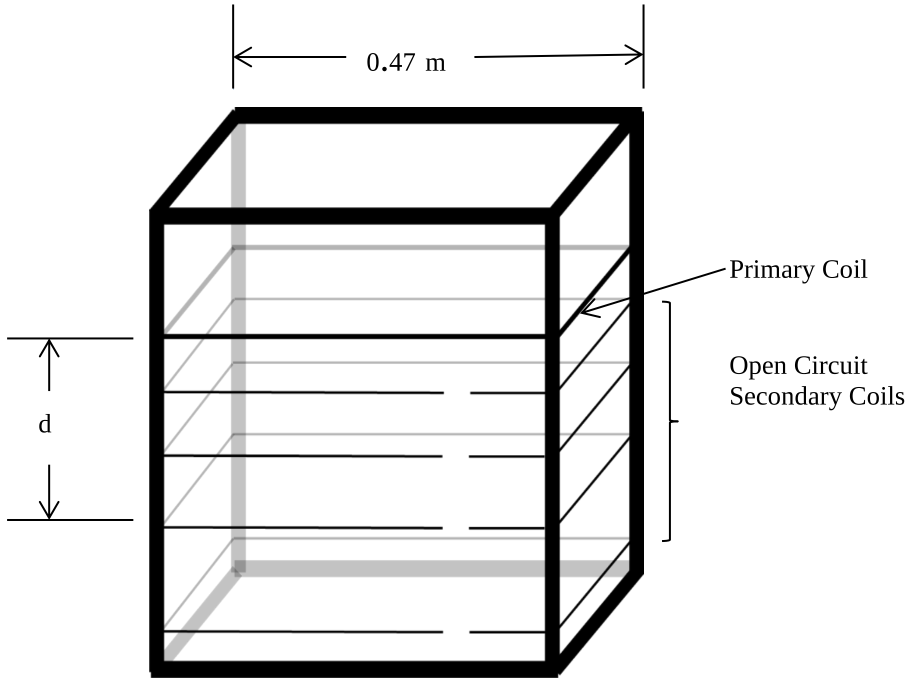

Figure 4. Square coil form, 0.47 m /side, used in induction

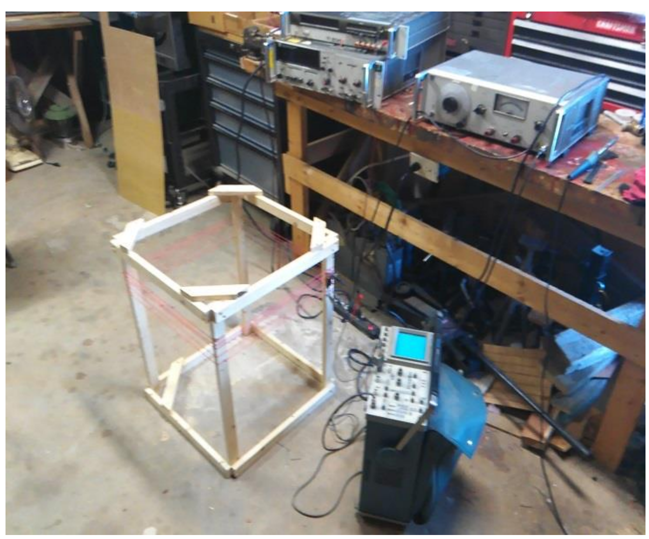

Figure 5. Picture of square coil form used for testing induction.
Also pictured is the test equipment employed in the experiment.

Table 2. Einsteinian Electrodynamics Induction Experiment, primary voltage = 8.0 v p-p.

Coil	**Distance d in.**	**Distance Avg. d mm**	**Measured V_{emf} v p-p**	**Computed V_{emf} v p-p**	**% Difference**
1	0.5	13.1	3.725	3.735	+0.27
2	1.0	25.4	2.925	2.903	-0.75
3	2.0	51.8	1.973	2.036	+3.19
4	3.0	77.5	1.517	1.577	+4.09

8.2 Discussion of Results

1. The results were close enough to support the induction formula of Eq. (20). This law is like the induction law of Neumann [2]. It is based on current elements instead of having to have flux-linkages of a closed coil.
2. Because this induction law is supported, the other induction laws based on the same derivation as Eq. (20) are strongly supported, even though they are not tested here.
3. It supports the concept of a magnetic field which acts directly between the two current elements and allows the induction voltage to be calculated. The conventional method of representing magnetic flux and induction by flux linkages requires a closed circuit instead of an open circuit. Thus the experimental results are not possible by conventional means.
4. The reduction of inducted voltage in Coils 3 and 4, at the larger distances from the primary coil, appears to be real based on the accuracy of the measurements. One possible cause of this error maybe due to the 60 pf input capacitance of the measuring multi-meter which creates a small load on the open circuit coils.

8.3 Experiment with induction with two round coils

Two identical round single turn solenoid coils 15.88in. dia. (0.4032 m) were wound on flat cardboard cake plates. (see Figures 6 & 7). The inductance of one of the coils was measured to be 1.5486 µH. For a computed inductance match, a *ds* = to 8.68 mm was required. Stranded wire was used. For the ds length to have zero stored energy (zero inductance), it was found that 64-81 sub-wires were required in the current element computation. Coil 2 was made primary and Coil 1 was placed 40.04 mm and 107.61 mm directly above Coil 2 in two tests. A 4733 pf capacitor was connected in parallel with Coil 2 and the test oscillator was connected directly to the parallel LC circuit of Coil 2. Resonance was achieved by varying the frequency of the test oscillator. A 4.7480 v RMS was applied to Coil 2. The RMS voltage was measured on the open terminals of Coil 1. These RMS voltages were scaled to represent having 8.00 v p-p on primary Coil 2. The corresponding v p-p on secondary Coil 1 is presented in Table 3 below. Computed values were computed using Eq. (22), (23), and (26).

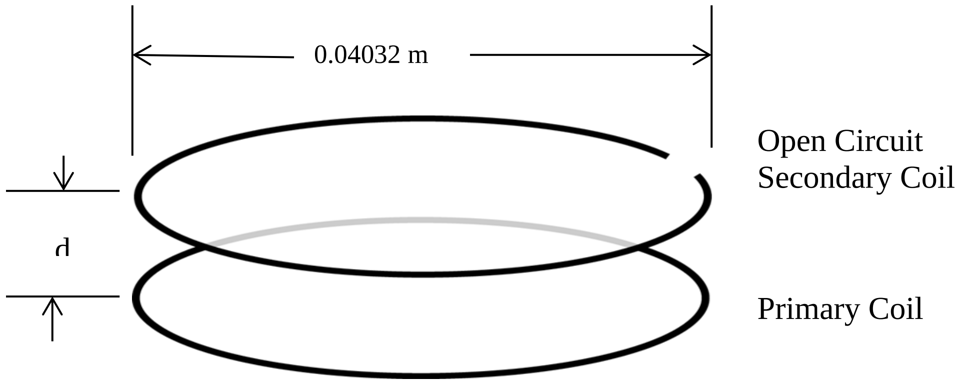

Figure 6. Round coils for induction experiment.

Table 3. Measured and Computed Induced Voltage in Round Coil 1.

Coil 1	**Distance mm**	**Measured v p-p**	**Computed v p-p**	**% Difference**
Position 1	40.04	2.101	2.106	+0.26
Position 2	107.61	1.042	1.090	+4.59

Figure 7. Picture of the two round coil forms employed in an experiment with induction.

8.4 Discussion of results

1. Again the induction formula of Eq. (20) is supported.
2. This was confirmed in computing the inductance with finite length of current elements that in order to properly set the limits of the piece-wise computer integration, every current element of Coil 2 must relate to every current element of Coil 1 only once in the computation.
3. The larger error (+4.59 %) occurring at the larger distance between the computed value and measured value corresponds to the error (+4.09%) that is shown in Table 2. As explained Section 8.2, Item 4, this error may be due in part to the 60 pf input capacitance of the measuring multi-meter.

9. Notes on the generated computer programs:

Seven short Borland Turbo Pascal programs were written for this paper. They are listed below. RMKS system of units is employed in all cases.

1. CAP_CAL.PAS Used for capacitor calibration.
2. L_CAL.PAS Used for calculating the "measured" inductance of a coil using resonance techniques.
3. IND_SQ3.PAS Used to calculate the inductance of the square one-turn coil.
4. CURELEMT.PAS Used to indicate zero energy storage or zero inductance in the length of a current element.
5. INDUCTN1.PAS Used to calculate the induced voltage on a single-turn square coil from varying current in another single-turn square coil.
6. IND_SOL4.PAS Used to calculate the inductance of a single-turn circular coil.
7. IND-SOL5.PAS Used to calculate the induced voltage on a single-turn circular coil from varying current in another single-turn circular coil.

10. Some Implications of Einsteinian Electrodynamics and Experiments

Some possible implications in the fields of physics and electrical engineering as result of this study: 1. the energy distribution in a current element, 2. the concept of a magnetic field, 3. the laws of physics relating to force between current elements, 4. radio wave propagation, 5. sub-atomic nuclear-type orbits, and 6. gravity fields. These are discussed in order below:

10.1 Energy distribution in a current element

When the current element is viewed as a group of parallel wires with current in the same direction, the energy distribution stored in the current element due to a current flowing in it can be discussed. Parallel wires with current in the same direction attract each other, so that there is a "pull" of molecules across the width of the current element that stores energy. It is known from the classical Ampère's Law that there is "repulsion" in the molecules along the length of the current element. When these two sources of energy storage become balanced, the correct length of the current element is obtained for use in our calculations for inductance with that law, where the self-inductance effect of the current element is omitted. When a current element is subdivided into a group of parallel wires with sub-current elements, then the integral of Eq. (18) may be employed to model the energy distribution in it. Using this technique and the integral of Eq. (6), it may be possible to predict conditions for generating instabilities in plasma.

So what happens in a physical situation where a current element is shorter than required for energy balance and the current element is composed of plasma? This situation occurs in the field of arc welding. Plasma ions are made to flow across the gap at the end of the welding rod to the surface of metal being welded. As a result the electrons are being pulled and combined with positive ions releasing energy, all due to Ampère’s Law. This is probably where the high intensity light and heat of welding comes from. Also, more energy is possibly released than required to promote the current flow. “Burning” of the metal may produce excess energy!

10.2 The concept of a magnetic field

The concept of a magnetic field as currently used in physics and electrical engineering is an artifact. It is an aid for handling gross conditions like induction in large coils, transformers, induction in motors, etc. On a smaller scale it can be seen that the magnetic field of a current element is due to a compressed or reduced electric field intensity **e** due to special relativity considerations. This is the field as seen by the observer, but not seen by the moving charge that produces the compressed or reduced **e**-field. The magnetic field depicted in a current element by the Biot-Savart Law, Eq. (33) is incorrect when applied to a current element in a wire carrying current. This is a result of the vector cross product term that shows no magnetic field in the direction of the flowing electron current. It is incorrect because the wire current is composed of discreet electrons, each having an electric field according to Eq. (1), and that there is a magnetic field in the direction of the flowing current. The artifact concept of the magnetic field that James Maxwell employed in his equations is the same as used by Faraday to account for large scale induction. The Maxwell equations should be reviewed with the concept of a magnetic field presented by Eq.(1). While vector analysis math is very useful in the field of electrodynamics, the employment of the vector cross product should be avoided in the field. This is because its use generally leads to the violation of Newton’s Third Law. An example of the mathematical complications involving the Lorentz Force Law and the use of the vector cross product is given in Prykarpatski’s paper [17]: “Classical Electromagnetic Theory Revisiting: The A. M. Ampère Law and the Vacuum Field Theory Approach”.

10.3 The laws of physics relating to magnetic force between current elements and charges

Lorentz Force Law, Eq. (2) is misleading, because it is assumed by many physicists that the source of the e-field and the magnetic field, as presented in the formula, can emanate from a single moving charge. The formula can apply to an electron beam moving in an e-field and a separate magnetic field in a cathode ray tube. The second term in the Lorentz Force Law involving the magnet field is also misleading. Lorentz/Grassmann’s Law Eq. (36) should be abandoned because it is based on the incorrect Biot-Sarvart law and violates Newton’s Third Law. The “magnetic field” depicted by the Biot-Savart Law, Eq. (33) is incorrect when applied to a current element in a wire carrying current. Ampère’s Law Eq. (17) and Eq. (18) should be reinstated in physics.

10.4 Radio Wave Propagation

The dynamic formula of Eq. (11) appears to be a 'natural' for radio wave propagation; it is repeated here for easy reference:

$$d\mathbf{F}_{12} = d\left(d\mathbf{f}_{12}\right) + d\mathbf{f}_{\mathbf{12}} = \frac{kq_2\sigma ds_1}{{r_{12}}^2}\frac{v^2}{c^2}\hat{\mathbf{r}}_{\mathbf{12}}\left(1\text{-}1.5\cos^2\theta\right)\left(1+\frac{2ra}{vc}\right) \tag{11}$$

While Eq. (11) describes a force between a charge and a current element, it can also describe a magnetic field at a distance from the current element. An antenna is a wire that carries current and emits radiation. Therefore, implicit in Eq. (11) is radio wave propagation. By varying the velocity of the electrons (varying current flow in the wire or antenna) a varying magnetic field should appear outside the antenna. And this should be the varying magnetic field that propagates throughout space at the speed of light. The following is a possible derivation of a propagation formula. It assumes a transmitting antenna being a vertical wire protruding from the earth's surface. Taking the q_2 and the "steady-state" term out of Eq. (11) we have an equation for the magnetic field $\mathbf{h}_\phi$ at a distance $\mathbf{r_{12}}$. Recall that a field is defined as force per unit charge.

$$d\mathbf{h}_\phi = \frac{kI_1ds_1}{{r_{12}}^2}\frac{1}{c^2}\mathbf{r}_{\mathbf{12}}\left(1\text{-}1.5\cos^2\theta\right)\left(\frac{2ra}{c}\right) \tag{45}$$

Noting that $I_1ds_1/v_1 = \sigma_1 ds_1$, $a = \frac{dv_1}{dt} = \frac{dI_1}{dt}\frac{1}{\sigma_1}$, letting $I_1 = I_{\max}\sin(wt)$ $\frac{dI_1}{dt} = I_{\max}w\cos(\omega t)$,

and inserting r/c in the cos term for correct phase representation:

$$d\mathbf{h}_\phi = \frac{\omega\mu_o I_1 ds_1}{c4\pi\sigma_1 r}\hat{\mathbf{r}}_{12}\left(1 - 1.5\cos^2\theta\right)I_{1\max}\cos\left[\omega(t-r/c)\right] \tag{46}$$

Compare Eq. (46) to an equation from a radio wave propagation text book [18]:

$$H_\phi = \frac{\omega\mu_o Ids}{c4\pi r}\sin\theta\cos\left[\omega(t-r/c)\right] \tag{47}$$

Since the antenna is a short vertical wire, then the θ terms represent the field lobes in the vertical plane. There is strong similarity between Eq. (46) and (47).

Eq. (43) illustrates the basic relativistic nature of radio wave propagation and field patterns. This is due to length contraction of the space surrounding the basic electron charge when it's movement is seen by a non-moving point. There are many factors involved in radio wave propagation. However, based on successful experiments with the classical Ampère's Law, Eq.(46) should have some validity and it may be one of the foundations in radio wave propagation. Also, it is an example of retarded action at a distance versus instantaneous action at a distance.

A test could be made comparing the vertical lobe structures presented by Eq. (46) and (47).

10.5 Sub-atomic nuclear-type orbits

The keystone equation, Eq. (38), can be applied to sub-atomic orbits in the manner of atomic Bohr orbits. It comprises both the electric and magnetic fields and can simplify the calculations by not having to treat the electric and magnetic fields separately. When the velocity vector of Eq. (38) is at right angles to the radial vector, the equation simplifies to having one gamma term in the numerator. This makes possible Bohr like analysis of sub-atomic or nuclear-type orbits where the speed of orbiting particles may approach the speed of light c [19].

10.6 Gravity fields

Eq. (1) when applied to a gravity field of a massive object allows for calculation in four dimensional spaces the relationship of two massive bodies moving relative to each other. They must be far enough apart that the gravity field can be considered emanating from a point in the center of the mass. Mercury appears to be far enough away from the sun for such a calculation.

It appears that a gravity field between two masses responds to their relative velocity as does the electric field between charges in current elements to their relative velocity. The perihelion advance of Mercury for 100 years was calculated this way. It matched the perihelion advance within 1% calculated by General Relativity [20].

For slowly relatively moving massive bodies the formula is:

$$\mathbf{F}_{12} = \hat{\mathbf{r}}_{12} \frac{Gm_1m_2}{r^2}\left[1+\frac{v^2}{c^2}\left(1-\frac{3}{2}\cos^2\theta\right)\right] \tag{48}$$

For fast relatively moving bodies, the formula is:

$$\mathbf{F}_{12} = \hat{\mathbf{r}}_{12} \frac{Gm_1m_2}{r^2}\frac{1}{\gamma\left[1-(v^2/c^2)\sin^2\theta\right]^{3/2}} \tag{49}$$

where γ, θ, and v are defined as in Eq. (1).

These are steady-state formulas. Dynamic equations may be applicable on some occasions such as for gravity waves. Eq. (48) and (49) may have a relation to the existence of dark matter.

11. Concluding Remarks

This paper demonstrates that electrodynamics has a theoretical basis, that of SR, which is derived from two postulates: the equivalence of physical laws in all inertial frames and the constancy of the speed of light for all observers.

The Einsteinian Electrodynamics laws when applied to slow relative velocities are very similar to the Ampère-Neumann Electrodynamics of Peter Graneau, Gauss's Law, and Weber's equation. The Ampère-Neumann Electrodynamics is useful at slow relative velocities just as the Newton laws appear as a limit in special relativity at slow relative speeds. The Einsteinian Electrodynamics formulas, Eq. (1) and Eq. (38) can be useful at high relative velocity speeds and therefore are the more general of the electrodynamics laws. SR Electrodynamics can rightly be called Einsteinian Electrodynamics.

It can be asked why does the slow electron velocity in current in a wire (on the order of cm's/sec) have such a large relativist effect. The answer is that there are a very large number of electrons in the wire.

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Notes

1. Updated subscripts and some formula 10/6/22. JK