Experiment with Ampere's Law and the Current Element

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The reason that a finite-length current element may legitimately be used in the mathematics of calculating forces and energy between current-carrying wires is that the current element contains canceling stored energy. This fact is demonstrated by an experiment conducted by the author. The inductance of several differently shaped single-turn coils was measured. The measured value of inductance was then compared to the value of inductance calculated employing the integral of Ampère's Law. It was discovered that, by adjusting the length of the current element, a match between the measured and calculated inductance could be obtained for all the single turn coils. The inductance of the current element itself was then calculated using the integral of Ampère's Law. The inductance of the length required for the match was found to be zero. There are mechanisms in the current element that give up energy to supply the energy stored in it so that the net stored energy is zero.

Introduction

Ampère's Law [1], expressed in RMKS units, (1) or (2) describes a force between current elements. A current element may be defined as a current carrying wire of small length, ds.

$$\mathbf{f_{12}} = -\hat{\mathbf{r}}_{12} \frac{kI_1 I_2 ds_1 ds_2}{c^2 r_{12}^{\ 2}} \left[2d\hat{\mathbf{s}}_1 \cdot d\hat{\mathbf{s}}_2 - 3(d\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2 \cdot \hat{\mathbf{r}}_{12}) \right] \quad (1)$$

where $\mathbf{f}_{12}$ is the force on current element 1 caused by current element 2, and $\mathbf{r}_{12}$ is the relative vector displacement from 1 to 2, *i.e.*, $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$. I_1 and I_2 are current magnitudes in current elements 1 and 2 respectively. The constants are $k = 1/4\pi\varepsilon_0$ (ε_0 = permittivity of free space) and $c =$ speed of light. Also Ampère's Law [2] can be expressed in term of angles as:

$$\mathbf{f_{12}} = -\hat{\mathbf{r}}_{12} \frac{kI_1 I_2 ds_1 ds_2}{c^2 r_{12}^{\ 2}} \left(2\sin\theta_1 \sin\theta_2 \cos\eta - \cos\theta_1 \cos\theta_2 \right) \quad (2)$$

where the angles are: $\theta_1 =$ angle between $d\mathbf{s}_1$ and $\mathbf{r}_{12}$; $\theta_2 =$ angle between $d\mathbf{s}_2$ and $\mathbf{r}_{12}$; $\eta =$ angle between the plane of $d\mathbf{s}_1$ with $\mathbf{r}_{12}$ and the plane of $d\mathbf{s}_2$ with $\mathbf{r}_{12}$. Eqs. (1) and (2) are equivalent, but (1) is mathematically easier to apply using vector analysis.

Eq. (1) may be integrated [3] as in (3) to arrive at a Eq. (4) for energy stored between current elements:

$$E_{12} = -\int_{\infty}^{r} \mathbf{f_{12}} dr \quad (3)$$

$$E_{12} = -\frac{kI_1 I_2 ds_1 ds_2}{c^2 r_{12}} \left[2d\hat{\mathbf{s}}_1 \cdot d\hat{\mathbf{s}}_2 - 3(d\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2 \cdot \hat{\mathbf{r}}_{12}) \right] \quad (4)$$

The angle terms as in (2) can be considered to be constant and brought outside the integral sign in (3) to make the integration simple. (4) can be expressed as:

$$E_{12} = \tfrac{1}{2} I^2 dL_{12} \quad (5)$$

where $I_1 = I_2$ as in a coil. The incremental inductance dL_{12} due to E_{12} may be calculated by the remaining terms in (4). Then the total inductance of the coil L_c may be calculated by adding all the incremental dL's from all the current element relationships in the coil.

$$L_c = \sum dL\text{'s} \quad (6)$$

This suggests an experiment to test Ampère's Law where the inductance L_m of a single turn coil is measured by precise electrical techniques. This technique involves measuring the resonant frequency of the coil with known capacitor and calculating L_m. The total energy stored in the coil is given by:

$$E_t = \tfrac{1}{2} I^2 L_m \quad (7)$$

So L_c and L_m can be compared directly, the current *I* through the coil being arbitrary. The inductance L of the coil is based on its physical characteristics. This makes a great experiment.

Even though Ampère's Law was determined by experiment around 1822, long before Einstein's Relativity, Ampère's Law is mathematically derivable from Special Relativity Theory, SRT [4]. Grassman's force law for current elements is wrong. It is a combination of Lorentz force law and the Biot-Savart law. Grassman's force law cannot be applied in any way that makes sense in the above experiment. Both the Lorentz force law and the Biot-Savart law must be questioned for validity since they comprise the Grassman's force law. However, that is not the present author's purpose of this paper.

Experiment

The inductance of several coils were both measured and calculated. The physical description of the coils is given in Table 1.

Table 1. Description of Coils in Experiment.

name	size:	wire diameter
circle	r = 10 inches	0.635 mm
		0.254 mm
		2.134 mm
square	side = 47 cm	0.635 mm
triangle	side = 47 cm	0.635 mm
3-D square	2 sides = 47 cm	
	2 sides = 47/cos 45°cm	0.635 mm
3-D tetra	4 sides = 26 inches	0.635 mm

The measuring techniques and the math involved in the calculations are both long and involved, and are not presented in this paper. They are, however, available from the author.

Table 2. Measured and Calculated Results, (for wire dia. = 0.635 mm)

Name:	ds (mm):	L_m (μhenries)	L_c (μhenries)	Δ %
Triangle	2.4	1.914819	1.909496	−0.28
Circle	2.7	2.399168	2.396962	−0.09
Square	2.8	2.717203	2.717502	0.01
3-D Square	3.2	3.222207	3.223535	0.04
3-D Tetra	3.1	3.760989	3.765264	0.11

Table 2 shows that the appropriate current element length is a weak function of the overall inductance of the coil, as well possibly other external parameters not associated with the internal parameters of the current element. Varying the ds length on either side of the ones presented in Table 2 will produce a calculated value of L_c that will be above or below the measured value of L_m. Since no self-inductance relationship of ds was employed in the calculation of L_c, it is deduced quite appropriately from Table 2 that for the measured and calculated values to agree so closely, the internal inductance of the current element ds must be zero.

Table 3. Circle coil Wire diameter as a Parameter.

wire diameter (mm)	L_m (μhenries)	ds (mm) required for L_c match	ds (mm) calculated for zero inductance in current element model
0.254	2.747	1	1.08
0.635	2.399	2.7	2.71
2.134	1.944	12	9.1

A computer model approximating the current element, ds, was developed that would calculate its inductance. Again the integral of Ampère's Law was employed for the calculation. A square cross-section of the wire was used to approximate the real round cross-section of the wire, areas being made equal. This was done for ease of applying the math to the calculations. The cross-section area was divided into 49 square 'wires' with all currents in the same direction. Then each of the 'wires' was divided in ds sub current elements half the length of the side of the cross-section of each of the 'wires'. The number of these sub current elements was varied, and thus the length of ds, to produce approximately zero inductance in the current elements. The right number of these sub current elements to produce approximately zero inductance in the current element model was found to be 135. These parameters were not varied from one wire diameter to another used in the computations. The results of the calculations are compared with the determined values of ds as in Table 2 for the circle coil when the wire diameter size of the coil was varied. Table 3 presents the results.

Measurements and calculations for ds lengths presented in Table 3 were done with sufficient accuracy to confirm that for an appropriate current element length, the inductance to be used in Ampere's Law calculations is zero. This is indeed fortunate. It validates Ampère's Law as well as ds usage in calculations.

Discussion

Successive sub current elements in the length dimension of the model of the overall current element store energy by repulsion as can be observed from Ampère's Law. That means that the parallel sub current elements in the model with parallel currents in the same direction give up energy to make the total in the overall current element zero. Therefore, if a gap much smaller than the overall size of the computed current element is employed and the current conduction is made by ions or such flexible conducting material, then ion current would act as a catalyst (a pull of the electron and proton nucleus together) to possibly release energy from the reaction greater than the amount supplied by the current producing power supply. This may happen in arc welding.

For microcircuit applications, it is observed in Table 3 that very smaller wires have higher inductance. An up-current small wire placed very near to a down-current wire would achieve higher inductance in a small space. A current element of zero inductance can be made longer by using larger wire diameter.

Conclusions

1. The experiment validates the use of Ampère's Law and the current element ds.
2. The 3-D coil experiments validate all the angle functions appearing in (2).
3. The experiment also is another validation of SRT.
4. There are possible applications for energy release in conductor gaps smaller than the zero-stored-energy-current-element length, which contain flexible conductor material such as ions or plasma. Light energy release in arc welding and in lightening in thunderstorms may be examples of this effect.
5. Lager inductances may be designed into microcircuits.
6. Zero-inductance conductors can be created using large diameter wires.

References

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